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COLLEGE ENTRANCE
EXAMINATION PAPERS

IN

PLANE GEOMETRY

COMPILED BY

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PREFACE

TEACHERS who prepare pupils for college have long been resigned to the fact that many pupils of admitted skill and reliability fail sadly to live up to expectations in the college entrance examinations. Fear of the unknown and a general lack of confidence seem to paralyze their powers. Some gradually recover their wits, but too late to answer all the questions in the given time.

Although this difficulty can never be removed for all pupils, it is certain that a greater familiarity with the conditions pertaining to college entrance examinations will be of much help to both teachers and students. At least such is the experience of the compilers of this collection of admission examinations in geometry. They have found that pupils who have been drilled until accustomed to take as daily fare the formal and actual examinations of the colleges within the time limits set by the colleges, have produced, on the whole, far better results than those who have not had this training. The former have been inspired with a confidence which lasts, a confidence which is based on the knowledge that they have overcome daily for weeks the same bugbear which will confront them at the college. It may seem to some that such work is superficial, mere "cramming," but even the most prejudiced must be convinced on second thought that a pupil who can daily do problems of such diversity has gained not only more confidence, but also more power than the pupil who is accustomed to doing only " originals " for which a given proposition is a clue. Logic and experience have proved to the editors that the final step in the preparation of a pupil for college entrance requirements should be a thorough drill on examinations similar to those which he has to pass. It is almost impossible for a pupil to pass a Harvard Geometry examination, for instance, with a high percentage if he has not had some practice on similar examinations under a time limit.

This book contains practically all of the Harvard and Yale examinations of the last twenty years, as well as many from the Massachusetts Institute of Technology. The book is devoted in such large proportion to the examinations of these colleges because they admit by examination only, whereas most other colleges admit by certificate.

Charles A. Marsh, A.M.
Harrie J. Phipps, B.S.

May, 1911.

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REGULAR POLYGON

	SIDE		APOTHEM	AREA			AREA OF SEGMENT
	in Terms of Radius.	in Terms of Apothem.	in Terms of Radius.	in Terms of Radius.	in Terms of Apothem.	in Terms of Side.	of a Circle Subtended by a Side
Triangle	$R\sqrt{3}$ $1.732R$	$2A\sqrt{3}$ $3.464A$	$\frac{1}{2}R$ $.5R$	$\frac{3}{4}R^2\sqrt{3}$ $1.299R^2$	$3A^2\sqrt{3}$ $5.196A^2$	$\frac{1}{4}S^2\sqrt{3}$ $.433S^2$	$\frac{R^2}{12}(4\pi-3\sqrt{3})$ $.6142R^2$
Square	$R\sqrt{2}$ $1.414R$	$2A$ $2A$	$\frac{1}{2}R\sqrt{2}$ $.707R$	$2R^2$ $2R^2$	$4A^2$ $4A^2$	S^2 S^2	$\frac{R^2}{4}(\pi-2)$ $.2854R^2$
Pentagon	$\frac{1}{2}R\sqrt{10-2\sqrt{5}}$ $1.176R$	$2A\sqrt{5-2\sqrt{5}}$ $1.453A$	$\frac{1}{4}R(\sqrt{5}+1)$ $0.809R$	$\frac{5}{8}R^2\sqrt{10+2\sqrt{5}}$ $2.378R^2$	$5A^2\sqrt{5-2\sqrt{5}}$ $3.633A^2$	$\frac{S^2}{4}\sqrt{25+10\sqrt{5}}$ $1.72045S^2$	$\frac{R^2}{40}(8\pi-5\sqrt{10+2\sqrt{5}})$ $0.1527R^2$
Hexagon	R R	$\frac{2}{3}A\sqrt{3}$ $1.155A$	$\frac{1}{2}R\sqrt{3}$ $0.866R$	$\frac{3}{2}R^2\sqrt{3}$ $2.598R^2$	$2A^2\sqrt{3}$ $3.464A^2$	$\frac{3}{2}S^2\sqrt{3}$ $2.598S^2$	$\frac{R^2}{12}(2\pi-3\sqrt{3})$ $0.0906R^2$
Octagon	$R\sqrt{2-\sqrt{2}}$ $0.7654R$	$2A(\sqrt{2}-1)$ $0.8284A$	$\frac{1}{2}R\sqrt{2+\sqrt{2}}$ $0.9239R$	$2R^2\sqrt{2}$ $2.8284R^2$	$8A^2(\sqrt{2}-1)$ $3.314A^2$	$2S^2(\sqrt{2}+1)$ $4.8284S^2$	$\frac{R^2}{8}(\pi-2\sqrt{2})$ $0.03915R^2$
Decagon	$\frac{R}{2}(\sqrt{5}-1)$ $0.618R$	$\frac{2A}{5}\sqrt{25-10\sqrt{5}}$ $0.649A$	$\frac{1}{4}R\sqrt{10+2\sqrt{5}}$ $0.9511R$	$\frac{5}{4}R^2\sqrt{10-2\sqrt{5}}$ $2.939R^2$	$2A^2\sqrt{25-10\sqrt{5}}$ $3.248A^2$	$\frac{5}{2}S^2\sqrt{5+2\sqrt{5}}$ $7.690S^2$	$\frac{R^2}{40}(4\pi-5\sqrt{10-2\sqrt{5}})$ $0.020272R^2$
Dodecagon	$R\sqrt{2-\sqrt{3}}$ $0.5176R$	$2A(2-\sqrt{3})$ $0.536A$	$\frac{1}{2}R\sqrt{2+\sqrt{3}}$ $1.029R$	$3R^2$ $3R^2$	$12A^2(2-\sqrt{3})$ $3.216A^2$	$3S^2(2+\sqrt{3})$ $11.196S^2$	$\frac{R^2}{12}(\pi-3)$ $0.0118R^2$

In the preparation of this table, the authors are greatly indebted to E. G. Hapgood, Headmaster of the Girls' Latin School, Boston.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO HARVARD

JUNE, 1890

1. Prove that if two sides of a triangle are respectively equal to two sides of another, but the included angle in the first triangle is greater than the included angle in the second, the third side of the first triangle is greater than the third side of the second.

If the included angle in the case of the first triangle is twice as great as the included angle of the other, is the third side of the one twice as long as the third side of the other?

2. Prove that through three points not in the same straight line one circumference can be drawn and only one.

A certain equilateral triangle has sides $8\sqrt{3}$ inches long; what is the radius of the circumference circumscribed about this triangle?

3. Prove that an angle which has its vertex within a circumference is measured by half the sum of the two arcs intercepted between its sides when extended in both directions.

The vertices of a quadrilateral inscribed in a circle divide the circumference into arcs which are to each other as 1, 2, 3, and 4. Find the angles between the opposite sides of the quadrilateral.

4. Prove that the diagonals and the line which joins the middle points of the parallel sides of a trapezoid meet in a point.

5. The radius of a certain circle is 9 inches; find the area of that one of all the regular polygons inscribed in it which has the shortest perimeter. How long a perimeter can a regular polygon inscribed in this circle have?

JUNE, 1891

(*In solving problems use for* π *the approximate value* $3\frac{1}{7}$.)

1. Prove that in an isosceles traingle the angles opposite the equal sides are equal to each other.

The area of a certain isosceles triangle is 50 square feet and each of its equal sides is 10 feet long; find the angles of the triangle.

2. Prove that in any quadrilateral circumscribed about a circle the sum of two opposite sides is equal to the sum of the other two opposite sides.

One vertex of a circumscribed quadrilateral and the directions of the sides which meet at the vertex are given; what is the locus of the centre of the inscribed circle?

3. From a fixed point O of a given circumference are drawn two chords OP, OQ, so as to make equal angles with a fixed chord, OR, between them. Prove that PQ will have the same direction whatever the magnitude of the equal angles.

4. The diagonals of a certain trapezoid which are 8 and 12 feet long, respectively, divide each other into segments which in the case of the shorter diagonal are 3 feet and 5 feet long. What are the segments of the other diagonal?

5. Assuming that as the number of sides of a circumscribed polygon is indefinitely increased, the perimeter approaches as a limit the circumference of the circle, and the area of the polygon the area of the circle; prove that the area of a circle is numerically equal to one-half the product of its radius by its circumference.

6. A regular hexagon, the perimeter of which is 42 inches, is inscribed in a circle; what is the area of this circle?

JUNE, 1892

(In solving problems use for π *the approximate value* $3\frac{1}{7}$.*)*

1. Prove that if two sides of a triangle are unequal, the angle opposite the greater side is greater than the angle opposite the less side.

In a certain right triangle one of the legs is half as long as the hypotenuse; what are the angles of the triangle?

2. Show how to find on a given indefinitely extended straight line in a plane, a point O which shall be equidistant from two given points A, B, in the plane. If A and B lie on a straight line which cuts the given line at an angle of 45° at a point 7 inches distant from A and 17 inches from B, show that OA will be 13 inches.

3. Prove that an angle formed by a tangent and a chord drawn through its point of contact is the supplement of any angle inscribed in the segment cut off by the chord. What is the locus of the centre of a circumference of given radius which cuts at right angles a given circumference?

4. Show that the areas of similar triangles are to each other as the squares of the homologous sides.

5. Prove that the square described upon the altitude of an equilateral triangle has an area three times as great as that of a square described upon half of one side of the triangle.

6. Find the area included between a circumference of radius 7 and the square inscribed within it.

JUNE, 1893

(*In solving problems use for* π *the approximate value* $3\frac{1}{7}$.)

1. Prove that two oblique lines drawn from a given point to a given line are equal if they meet the latter at equal distances from the foot of the perpendicular dropped from the point upon it.

How many lines can be drawn through a given point in a plane so as to form in each case an isosceles triangle with two given lines in the plane?

2. Prove that in the same circle, or in equal circles, equal chords are equally distant from the centre, and that of two unequal chords the less is at the greater distance from the centre.

Two chords of a certain circle bisect each other. One of them is 10 inches long; how far is it from the centre of the circle?

A variable chord passes, when produced, through a fixed point without a given circle. What is the locus of the middle point of the chord?

3. A common tangent of two circumferences which touch each other externally at A, touches the two circumferences at B and C respectively; show that BA is perpendicular to AC.

4. Assuming that the areas of two triangles which have an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles, prove that the bisector of an angle of the triangle divides the opposite side into parts which are proportional to the sides adjacent to them.

5. Prove that the circumferences of two circles have the same ratio as their radii.

6. A quarter-mile running track consists of two parallel straight portions joined together at the ends by semicircumferences. The extreme length of the plot enclosed by the track is 180 yards. Find the cost of sodding this plot at a quarter of a dollar per square yard.

JUNE, 1894

(*In solving problems use for* π *the approximate value* $3\frac{1}{7}$.)

1. Prove that any quadrilateral the opposite sides of which are equal, is a parallelogram.

A certain parallelogram inscribed in a circumference has two sides 20 feet in length and two sides 15 feet in length; what are the lengths of the diagonals?

2. Prove that if one acute angle of a triangle is double another, the triangle can be divided into two isosceles triangles by a straight line drawn through the vertex of the third angle.

Upon a given base is constructed a triangle one of the base angles of which is double the other. The bisector of the larger base angle meets the opposite side at the point P. Find the locus of P.

3. Show how to find a mean proportional between two given straight lines, but do not prove that your construction is correct.

Prove that if from a point, O, in the base, BC, of a triangle, ABC, straight lines be drawn parallel to the sides, AB, AC, respectively, so as to meet AC in M and AB in N, the area of the triangle AMN is a mean proportional between the areas of the triangles BNO and CMO.

4. Assuming that the areas of two parallelograms which have an angle and a side common and two other sides unequal, but commensurable, are to each other as the unequal sides, prove that the same proportion holds good when these sides have no common measure.

5. Every cross-section of the train house of a railway station has the form of a pointed arch made of two circular arcs the centres of which are on the ground. The radius of each arc is equal to the width of the building (210 feet); find the distance across the building measured over the roof, and show that the area of the cross-section is $3675\,(4\pi-3\sqrt{3})$ square feet.

SEPTEMBER, 1894

One question may be omitted.

(*In solving problems use for* π *the approximate value* $3\frac{1}{7}$.)

1. Prove that any quadrilateral the diagonals of which bisect each other is a parallelogram.

The diagonals of a parallelogram circumscribed about a circumference are 60 inches and 80 inches long respectively. How long are the sides?

2. Prove that the difference of the angles at the base of a triangle is double the angle between a perpendicular to the base and the bisector of the vertical angle.

The sum of the base angles of each of a number of triangles constructed on a given base 10 inches long is 150°. What is the locus of the vertices of these triangles.?

3. Show how to find a fourth proportional to three given lines, but do not prove that your construction is correct.

One circle touches another internally at O, and a chord AB of the larger circle touches the smaller one at C. Prove that AO makes with the common tangent to the circles an angle equal to ABO and that CO bisects the angle AOB. State without proof some relation that exists between the lines AO, CB, BO, and AC.

4. Assuming that the areas of two rectangles which have equal altitudes are to each other as their bases when the latter are commensurable, show that the same proportionality exists when the bases have no common measure.

5. A kite-shaped racing track is formed by a circular arc and two tangents at its extremities. The tangents meet at an angle of 60°. The riders are to go round the track, one on a line close to the inner edge, the other on a line everywhere $5\frac{1}{4}$ feet outside the first line. Show that the second rider is handicapped by about 22 feet.

JUNE, 1895

One question may be omitted.

(*In solving problems use for* π *the approximate value* $3\frac{1}{7}$.)

1. Prove that if two straight lines are so cut by a third that corresponding alternate-interior angles are equal, the two lines are parallel to each other.

2. Prove that an angle formed by two chords intersecting within a circumference is measured by one-half the sum of the arcs intercepted between its sides and between the sides of its vertical angle.

Two chords which intersect within a certain circumference divide the latter into parts the lengths of which, taken in order, are as 1, 1, 2, and 5; what angles do the chords make with each other?

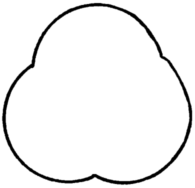
3. Through the point of contact of two circles which touch each other externally, any straight line is drawn terminated by the circumferences; show that the tangents at its extremities are parallel to each other.

What is the locus of the point of contact of tangents drawn from a fixed point to the different members of a system of concentric circumferences?

4. Prove that, if from a point without a circle a secant and a tangent be drawn, the tangent is a mean proportional between the whole secant and the part without the circle.

Show (without proving that your construction is correct) how you would draw a tangent to a circumference from a point without it.

5. Prove that the area of any regular polygon of an even number of sides ($2n$) inscribed in a circle is a mean proportional between the areas of the inscribed and the circumscribed polygons of half the number of sides. If n be indefinitely increased, what limit or limits do these three areas approach?



6. The perimeter of a certain church window is made up of three equal semicircumferences, the centres of which form the vertices of an equilateral triangle which has sides $3\frac{1}{2}$ feet long. Find the area of the window and the length of its perimeter.

SEPTEMBER, 1895

One question may be omitted.

(*In solving problems use for* π *the approximate value* $3\frac{1}{7}$.)

1. Prove that every point in the bisector of an angle is equally distant from the sides of the angle. State the converse of this proposition. Is this converse true?

2. Prove that an angle formed by two secants intersecting without a circumference is measured by half the difference of the arcs which the sides of the angle intercept.

A certain pair of secant lines which intersect without a circle divide the circumference into parts the lengths of which, taken in order, are to one another as 1, 2, 3, and 4. What angles do the lines make with each other?

3. Two given circles touch each other externally at the point P, where they have the common tangent PC. They are also touched by the line AB in the points A and B respectively. Show that the circle described on AB as diameter has its centre on PC, and touches at P the straight line which joins the centres of the two given circles.

4. Show how to describe upon a given straight line a segment which shall contain a given angle.

A and B are two fixed points on the circumference of a circle, and PQ is any diameter. What is the locus of the intersection of PA and QB?

5. C is any point on the straight portion, AB, of the boundary of a semicircle. CD, drawn at right angles to AB, meets the circumference at D. DO is drawn to the centre, O, of the circle, and the perpendicular dropped from C upon OD meets OD at E. Show that DC is a mean proportional to AO and DE.

State the fundamental theorem in the method of limits as used in Plane Geometry.

6. A horse is tethered to a hook on the inner side of a fence which bounds a circular grass plot. His tether is so long that he can just reach the centre of the plot. The area of so much of the plot as he can graze over is $\frac{98}{3}(4\pi - 3\sqrt{3})$ square rods; find the length of the tether and the circumference of the plot.

JUNE, 1896

One question may be omitted.

(In solving problems use for π the approximate value $3\frac{1}{7}$.)

1. Prove that if two oblique lines drawn from a point to a straight line meet this line at unequal distances from the foot of the perpendicular dropped upon it from the given point, the more remote is the longer.

2. Prove that the distances of the point of intersection of any two tangents to a circle from their points of contact are equal.

A straight line drawn through the centre of a certain circle and through an external point, P, cuts the circumference at points distant 8 and 18 inches respectively from P. What is the length of a tangent drawn from P to the circumference?

3. Given an arc of a circle, the chord subtended by the arc, and the tangent to the arc at one extremity, show that the perpendiculars dropped from the middle point of the arc on the tangent and chord, respectively, are equal.

One extremity of the base of a triangle is given and the centre of the circumscribed circle. What is the locus of the middle point of the base?

4. Prove that in any triangle the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides diminished by twice the product of one of those sides and the projection of the other upon that side.

Show very briefly how to construct a triangle having given the base, the projections of the other sides on the base, and the projection of the base on one of these sides.

5. Show that the areas of similar triangles are to one another as the areas of their inscribed circles.

The area of a certain triangle the altitude of which is $\sqrt{2}$, is bisected by a line drawn parallel to the base. What is the distance of this line from the vertex?

6. Two flower beds have equal perimeters. One of the beds is circular and the other has the form of a regular hexagon. The circular bed is closely surrounded by a walk 7 feet wide bounded by a circumference concentric with the bed. The area of the walk is to that of the bed as 7 to 9. Find the diameter of the circular bed and the area of the hexagonal bed.

SEPTEMBER, 1896

One question may be omitted.

(In solving problems use for π the approximate value $3\frac{1}{7}$.)

1. Prove that, if one of two convex broken lines which have the same extremities envelops the other, the first is the longer.

2. Prove that, when two circumferences intersect each other, the line which joins their centres bisects at right angles their common chord.

The centres of two circles of radii 8 inches and 6 inches respectively are 10 inches apart. Show that the common chord is 9.6 inches long.

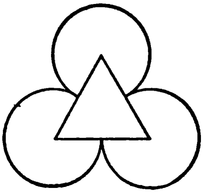
3. Show that, if two parallel tangents to a circle are intercepted by a third tangent, the part of the third tangent between the other two subtends a right angle at the centre of the circle.

State briefly how you might find a fourth proportional to three given straight lines.

4. Prove that in any obtuse-angled triangle the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides, increased by twice the product of one of these sides and the projection of the other upon that side.

What is the locus of the vertices of all the triangles so constructed on a given base that the radii of their circumscribed circles are all equal to a given line?

5. The line passing through the centres of two circles which touch each other externally at A, meets a common tangent, which touches the circles at B and C respectively, at the point S. Show that SA is a mean proportional between SB and SC.



6. The perimeter of a certain church window is made up of three equal circular arcs the centres of which are the vertices of an equilateral triangle. Each of the arcs subtends an angle of 300° at its own centre. Find the area of the window, assuming the length of the perimeter to be 110 feet.

JUNE, 1897

Take as many propositions as you can in the time allowed for the examination, attacking first those which seem to you least difficult. Do not prove that your answers or constructions are correct unless you are expressly asked to do so.

1. Prove that two triangles are equal when a side and the two adjacent angles of the one are respectively equal to a side and the two adjacent angles of the other.

2. Prove that an angle inscribed in a circle is measured by one half the arc intercepted between its sides.

Two tangents to a circle meet the circumference at points which divide it into two arcs one of which is twice as long as the other. At what angle do these tangents intersect each other?

3. Two equal circles touch each other externally. Through the point of contact two chords are drawn, one in each circle, at right angles to each other. Show that the straight line which joins their other extremities is equal to the diameter of either circle and parallel to the line which joins their centres.

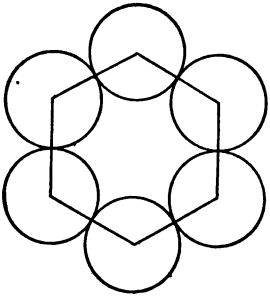
A circle is drawn so as to have a given line as chord. What is the locus of the middle point of the line which joins the centre of the circle with one extremity of this chord?

4. Prove that if a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

Show how to draw a line parallel to the base of a triangle which shall form with the other two sides produced a triangle having an area twice as great as that of the original triangle. Prove your construction to be correct.

5. Prove that if AB is a diameter of a circle and ACD a straight line which cuts the circumference in C and the tangent drawn at B in D, $\overline{AC}\cdot\overline{AD}=\overline{AB}^2$.

6. Six coins, each of radius a, are placed close together on a table so that their centres are at the vertices of a regular hexagon. Find the perimeter and the area of the enclosed figure.



SEPTEMBER, 1897

Take as many propositions as you can in the time allowed for the examination, attacking first those which seem to you least difficult. Do not prove that your answers or constructions are correct unless you are expressly asked to do so.

1. Prove that the sum of all the angles of any polygon is equal to twice as many right angles as the figure has sides less four.

2. Prove that in the same circle, or in equal circles, equal arcs are subtended by equal chords.

Two equal chords of a certain circle meet on the circumference at an angle of 60° and intercept an arc 8 feet long. How long an arc does each chord subtend?

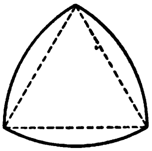
3. Prove that, if a circle be described on one of the sides of a right-angled triangle as diameter, the tangent drawn to it at the point where it cuts the hypotenuse bisects the other side.

Show how to describe a circle of given radius which shall touch a given circle externally and be tangent to a given straight line which intersects the given circle.

4. Prove that a perpendicular drawn from the vertex of the right angle to the hypotenuse of a right-angled triangle is a mean proportional to the segments of the hypotenuse.

What is the locus of the middle point of a side of a right-angled triangle the hypotenuse of which is a given line?

5. Two circles touch each other externally at A; a straight line touches the circles at B and C, and is produced to meet at S the straight line joining their centres. Prove that $\overline{SB}\cdot\overline{SC}=\overline{SA}^2$.



6. The perimeter of a certain grass plot is made up of three equal circular arcs each of which has its centre at a vertex of an equilateral triangle, the length of a side of which is a, and is subtended by the opposite side of the triangle. Show that the perimeter of the plot is πa and that its area is $\frac{1}{2}a^2(\pi-\sqrt{3})$.

NEW METHOD, JUNE, 1898

1. If two triangles have the three sides of one respectively equal to the three sides of the other, the triangles are equal.

2. Through three points not lying in a straight line one circumference, and only one, can be drawn.

If the points lie at the vertices of an equilateral triangle the length of a side of which is 3, what is the length of the radius of the circle?

3. Through one of the points of intersection of two circles draw a chord of one circle which shall be bisected by the other circle.

A set of triangles have a common base, and their vertices lie on a circumference, of which the base of the triangles is a chord. What is the locus of the point of intersection of the medial lines of one of the triangles?

4. Prove that, if a polygon inscribed in a circle is equiangular, it is also equilateral, provided that the number of sides is odd.

Is the theorem true when the number of sides is even?

5. Three radii are drawn in a circle of radius $2a$ so as to divide the circumference into three equal parts; and, with the middle points of these radii as centres, arcs are drawn, each with radius a, so as to form a closed figure (trefoil). Show that the length of the perimeter of the trefoil is equal to that of the circumference of the circle, and find its area.

6. If one of two parallel lines is perpendicular to a plane, the other is also perpendicular to the plane.

Can a plane always be drawn through a given point parallel to two straight lines in space that do not intersect each other? Can more than one such plane ever be drawn?

7. The lateral area of a cone of revolution is equal to the product of the circumference of the base by half the slant height.

8. The angle between two arcs of great circles is measured by the arc of a great circle described from its vertex as a pole and included between its sides.

9. The inner radius of a hemispherical bowl is $\sqrt{6}$ inches. Show that a cube 2 inches on an edge can just be placed under the bowl.

What portion of the earth's surface, assumed to be spherical, lies north of the sixtieth parallel of latitude? Does twice as much lie north of the thirtieth parallel?

OLD METHOD, JUNE, 1898

1. If two triangles have the three sides of one respectively equal to the three sides of the other, the triangles are equal.

2. Through three points not lying in a straight line one circumference, and only one, can be drawn.

If the points lie at the vertices of an equilateral triangle the length of a side of which is 3, what is the length of the radius of the circle?

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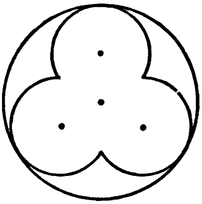
Is the theorem true when the number of sides is even?



5. The area of a circle is the limit which the areas of regular inscribed and circumscribed polygons approach when the number of their sides is increased indefinitely.

6. Three radii are drawn in a circle of radius $2a$ so as to divide the circumference into three equal parts; and, with the middle points of these radii as centres, arcs are drawn each with radius a, so as to form a closed figure (trefoil). Show that the length of the perimeter of the trefoil is equal to that of the circumference of the circle, and find its area.

OLD METHOD, SEPTEMBER, 1898

1. If two angles of a triangle are equal, the triangle is isosceles.

2. When two tangents to the same circle intersect each other, the distances from their point of intersection to their points of tangency are equal.

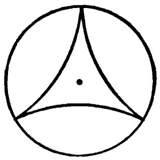
If these distances are each equal to 5 and the distance of the point of intersection of the tangents from the centre of the circle is 13, what is the length of the chord joining the points of tangency?

3. Triangles are constructed on a given line as base. What is the locus of the foot of the perpendicular dropped from the middle point of the base on either side? Prove your answer to be correct.

Construct a mean proportional to two given lines, and prove your construction to be correct.

4. Two circles with centres at O, O' are tangent to each other externally, and AB is one of the common tangents that do not pass through the point of contact of the circles. A circumference is described on OO' as diameter; show that it is tangent to AB.

5. Parallelograms having equal bases and equal altitudes are equivalent.



6. Three points divide a circumference into equal parts. Through each pair of these points an arc of a circle is described, tangent to the radii drawn to the points and lying wholly within the circle. Find the perimeter of the figure thus formed, and show that its area is $3(\sqrt{3}-\frac{1}{2}\pi)a^2$, where a denotes the radius of the circle.

NEW METHOD, JUNE, 1899

The University provides a Syllabus.

The order of propositions belonging to one and the same Book is not prescribed; but it is not expected that a proposition of a given Book shall be proved by the aid of propositions appearing in later Books. Should the candidate, however, have used a text-book in which the division into Books is inconsistent with the division of the Syllabus, and should he prefer to follow the order of propositions with which he is familiar, he will be allowed to do so on stating in his examination book the name of the text-book he has used.

Omit one of the starred questions.

* **1.** Prove that if two right triangles have the hypotenuse and a side of one respectively equal to the hypotenuse and a side of the other, the triangles are equal.

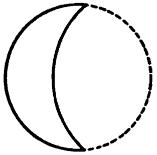
Two triangles have an angle, the opposite side, and another side of one equal respectively to an angle, the opposite side, and another side of the other. Are these triangles necessarily equal? Give roughly, by the aid of a figure, the reason for your answer without giving a formal proof.

* **2.** Prove that two parallel straight lines intercept equal arcs on a circumference.

A straight line joins the centres of two circles and cuts the first circle in the points A and B, and the second circle in the points C and D. Four parallel lines pass through A, B, C, and D, respectively. One-half of the first circumference lies between the first two of these lines. What part of the second circumference lies between the second two lines, and what angle do these four lines make with the line AB?

* **3.** A set of circles are tangent to a given line at a given point; and a second set of circles are tangent to a second line at the same point. What is the locus of the points of intersection of equal circles of the two families? Prove that your answer is correct.

4. Three unequal circles are so situated that each of them is externally tangent to the other two. At the points of contact tangents are drawn. Prove that these three tangents meet in a point.



* **5.** A crescent-shaped region is bounded by a semicircumference of radius a, and another circular arc whose centre lies on the semicircumference produced. Find the area and the perimeter of the region.

6. Prove that through any given straight line a plane can be passed perpendicular to any given plane.

Is it ever possible to pass more than one such plane through the line? If so, when? Can a plane always be passed through a given straight line perpendicular to a given line?

7. What is meant by the pole of a circle on a sphere?

Prove that all the points on the circumference of a circle on a sphere are equally distant from either of its poles.

Define the term "polar triangles."

8. AB, AC, AD are three edges of a cube which meet in the vertex A. A plane is passed through the middle points of these edges. If the cube contains 8 cubic feet, find the volume of the corner cut off by the plane, and the length of the perpendicular dropped from the centre of the cube on the plane.

OLD METHOD, JUNE, 1899

1. Prove that if two right triangles have the hypotenuse and a side of one respectively equal to the hypotenuse and a side of the other, the triangles are equal.

Two triangles have an angle, the opposite side, and another side of one equal respectively to an angle, the opposite side, and another side of the other. Are these triangles necessarily equal? Give roughly, by the aid of a figure, the reason for your answer without giving a formal proof.

2. Prove that two parallel straight lines intercept equal arcs on a circumference.

A straight line joins the centres of two circles and cuts the first circle in the points A and B, and the second circle in the points C and D. Four parallel lines pass through A, B, C, and D respectively. One-half of the first circumference lies between the first two of these lines. What part of the second circumference lies between the second two lines, and what angle do these four lines make with the line AB?

3. A set of circles are tangent to a given line at a given point; and a second set of circles are tangent to a second line at the same point. What is the locus of the points of intersection of equal circles of the two families? Prove that your answer is correct.

4. Three unequal circles are so situated that each of them is externally tangent to the other two. At the points of contact tangents are drawn. Prove that these three tangents meet in a point.

5. Construct the tangent to a given circle from a point without the circle; and prove the correctness of your construction.

6. A crescent-shaped region is bounded by a semicircumference of radius a, and another circular arc whose centre lies on the semicircumference produced. Find the area and the perimeter of the region.

OLD METHOD, SEPTEMBER, 1899

1. Prove that the diagonals of a parallelogram bisect each other.

One side of a parallelogram is produced in one direction. The opposite side is produced by the same amount in the opposite direction. Prove that the line joining the points thus reached passes through the point of intersection of the diagonals.

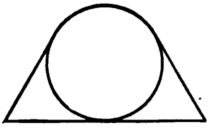
2. Prove that the area of a regular polygon is equal to half the product of its perimeter and its apothem.

Find the area of a regular hexagon each of whose sides is 1 foot long.

3. Lines are drawn parallel to the base of a triangle and are terminated by its sides. What is the locus of their middle points? Prove that your answer is correct.

4. Four points, A, B, C, D, lie on a circle with centre at O. A circle passes through A and C and touches the line OA; a second circle passes through B and D and touches the line OD. These circles intersect in E and F. Prove that the line EF produced passes through O.

5. Prove that in equal circles angles at the centres are in the same ratio as their intercepted arcs.



6. A straight rod lies on the deck of a vessel with its middle point resting against a mast 2 feet in diameter. A rope, one end of which is fastened to one end of the rod, passes behind the mast and after being drawn taut is fastened to the other end of the rod.

The rope makes at each end an angle of 60° with the rod. Find the length of the rod and the length of the rope.

NEW METHOD, JUNE, 1900

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Omit two of the starred questions.

* **1.** Prove that if two angles of a triangle are unequal, the side opposite the greater angle is greater than the side opposite the less angle.

In a quadrilateral $ABCD$ the lengths of the sides AB and BC are equal and the angle A is greater than the angle C. Which is longer, the side AD or the side CD? Give the reason for your answer.

* **2.** Prove that if the sum of two opposite angles of a quadrilateral is two right angles, the quadrilateral can be inscribed in a circle.

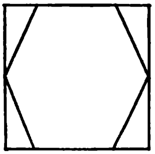
In a triangle ABC two points D and E are so taken on the sides AB and AC respectively that $AD:AC=AE:AB$. Prove that a circumference can be passed through the four points $BCDE$.

3. An angle of 60° moves so that both of its sides touch a fixed circle of radius 5 feet. What is the locus of its vertex? Prove your answer to be correct.

4. Two unequal triangles ABC and $A'B'C'$ have the sides AB, BC, CA of one respectively parallel to the sides $A'B'$, $B'C'$, $C'A'$ of the other. Prove that the lines AA', BB', CC' meet in a point.

*** 5.** Show how to divide a given straight line into five equal parts; and prove the correctness of your construction.

*** 6.** A square, each of whose sides is 6 inches long, has its corners cut off in such a way as to make it into an equilateral hexagon. What is the area of each of the corners?



7. Prove that planes perpendicular to the same straight line are parallel to each other.

Is it true that planes parallel to the same straight line are parallel to each other? Illustrate by a figure.

*** 8.** Prove that the plane passed through two diagonally opposite edges of a parallelopiped divides it into two equivalent triangular prisms.

9. When is a plane said to be tangent to a cylinder?

Prove that a plane passing through a tangent to the base of a circular cylinder and the element drawn through the point of contact is tangent to the cylinder.

10. A lead pencil $\frac{1}{4}$ of an inch in diameter is sharpened so that its end has the form of a right circular cone whose altitude is $\frac{5}{8}$ of an inch. If the pencil is resharpened in the same way beginning at a point $\frac{1}{8}$ of an inch further up the pencil, how much material will be removed.

NEW METHOD, SEPTEMBER, 1900

The order of propositions belonging to one and the same Book is not prescribed; but it is not expected that a proposition of a given Book shall be proved by the aid of propositions appearing in later Books. Should the candidate, however, have used a text-book in which the division into Books is inconsistent with the division of the Syllabus, and should he prefer to follow the order of propositions with which he is familiar, he will be allowed to do so on stating in his examination book the name of the text-book he has used.

Omit two of the starred questions.

* **1.** Prove that in an isosceles triangle the angles opposite the equal sides are equal.

Given two lines AB and BC. Show how to draw through a point D of AB a third line making with BC the same angle which AB makes with BC.

* **2.** Prove that an angle formed by a tangent and a chord is measured by one-half the intercepted arc.

What is meant here by the statement that an angle is "measured by" half an arc?

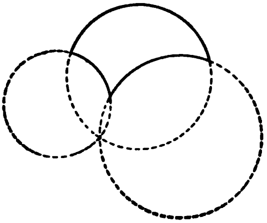
A certain diameter of a circle makes with a tangent an angle of 70°. Find the angles which the tangent makes with the chords which join the point of contact with the extremities of the diameter.

3. Through a point A on the circumference of a circle, chords are drawn. On each one of these chords a point is taken one-third of the distance from A to the other end of the chord. Find the locus of these points, and prove that your answer is correct.

4. Two circles intersect at the points A and B. Prove that they make equal angles with one another at A and at B.

(N. B.—By the angle between two circles at their point of intersection is meant the angle between their tangents at this point.)

A plane area (the upper area in the figure) is bounded by three arcs of circles which if produced would meet in a point. Prove that the sum of the angles between the circles at the vertices of this area is equal to two right angles.



*** 5.** Prove that the circumference of a circle is the limit which the perimeters of regular inscribed and circumscribed polygons approach when the number of their sides is increased indefinitely.

Find the perimeter of a regular hexagon inscribed in a circle of radius a; and also of a regular hexagon circumscribed about the same circle. What can you infer from these results concerning the value of π?

*** 6.** A square, each of whose sides is 5 inches long, has its corners cut off in such a way as to make it into a regular octagon. Find the area and the perimeter of the octagon.

7. Prove that two dihedral angles are equal if their plane angles are equal.

Is it true that if a plane is parallel to one of two parallel lines it is parallel to the other? Illustrate by a figure.

Is it true that if a line is parallel to one of two parallel planes it is parallel to the other? Illustrate by a figure.

*** 8.** Prove that the volume of a triangular prism is equal to the product of its base by its altitude.

9. Prove that a plane tangent to a sphere is perpendicular to the radius drawn to the point of contact.

10. What is the area in square inches of a spherical triangle drawn on a sphere of radius one foot, if the lengths in inches of the sides of the polar triangle are 3π, 2π, 2π.

OLD METHOD, JUNE, 1901

1. Prove that, if two angles of a triangle are equal, the triangle is isosceles.

The triangle *ABC* has a right angle at *A*. Through *A* a line is drawn forming with *AB* an angle equal to *B*, and cutting *BC* in *D*. Prove that the two triangles *ABD* and *ADC* are isosceles, and that *D* is the middle point of *BC*.

2. Prove that, when two tangents to the same circle intersect each other, the distances from their point of intersection to their points of contact are equal.

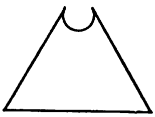
Two circles are tangent externally at *A*; *BC* is a common tangent, *B* and *C* being the points of contact. Prove that the angle *BAC* is a right angle.

Hint: Draw the common tangent at *A*.

3. Given a fixed point *D* within a triangle *ABC*. Choose any point, *E*, on the perimeter of the triangle, draw *DE*, and let *P* be the middle point of *DE*. Find the locus of *P* when *E* traces out the whole perimeter of the triangle. Describe the position of the locus exactly, and prove the correctness of your answer.

4. Two parallel tangents are drawn to a circle, and a third tangent is drawn intersecting these two in *A* and *B*, and tangent to the circle at *C*. Prove that the product of the segments *AC* and *CB* is equal to the square of the radius of the circle.

5. Given a straight line and two points, one of which lies on the line. Show how to construct the centre of a circle passing through the two points and tangent to the line at the given point of the line. Is there any case in which this construction is impossible?

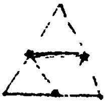


6. A piece is cut out of an equilateral triangle by means of an arc of a circle tangent to two sides. The side of the triangle is 7 inches and the radius of the circle 1 inch. Compute to two decimal places the perimeter and the area of the figure which is left.

SEPTEMBER, 1901

1. Prove that two triangles are equal if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

Prove that the two straight lines which join the middle points of the sides of an isosceles triangle to the middle point of the base are equal to each other.



2. Prove that an angle inscribed in a circumference is measured by one-half its intercepted arc.

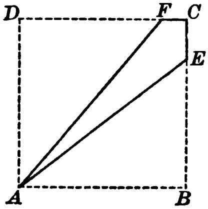
If through one of the points of intersection of two circumferences a diameter of each circle be drawn, prove that the straight line which joins the extremities of these diameters passes through the other point of intersection and is perpendicular to the common chord.

3. Given two intersecting straight lines AC and AB. Find the locus of a point which is twice as far from AB as from AC.

4. ABC is a triangle; BD, CE are lines drawn making equal angles with BC, meeting the opposite sides in D and E, and each other in F. Prove that, if the angle AFE is equal to the angle AFD, the triangle ABC is isosceles.

5. The triangle ABC has a right angle at C and the medians AD and BE are drawn. Prove that

$$5AB^2 = 4(BE^2 + AD^2).$$



6. Given a square $ABCD$. Show how to find two points *E* and *F* on *BC* and *CD* respectively, *CE* being twice the length of *CF*, such that the area of the quadrilateral *AECF* shall be one-fifth the area of the square.

JUNE, 1902

(One hour and a half.)

1. Prove that the sum of the three angles of any triangle is equal to two right angles.

Prove that lines drawn from a point within a triangle to the extremities of the base include an angle greater than the vertical angle of the triangle.

2. Prove that, if from a fixed point without a circle a secant is drawn, the product of the whole secant and its external segment is the same for all directions of the secant.

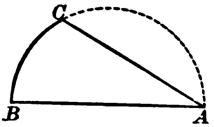
Prove that, if from any point on the common chord produced of two intersecting circles tangents are drawn to the two circles, the lengths of these tangents are equal.

3. Given a square *ABCD*. Let *E* be the middle point of *CD*, and draw *BE*. A line is drawn parallel to *BE* and cutting the square. Let *P* be the middle point of the segment of this line within the square. Find the locus of *P* when the line moves, always remaining parallel to *BE*. Describe the locus exactly, and prove the correctness of your answer.

4. Let *ABCD* be any parallelogram, and from any point *P* in the diagonal *AC* draw the straight line *PM*, cutting *AB* in *M*, *BC* in *N*, *CD* in *L*, and *AD* in *K*. Prove that

$$PM \times PN = PK \times PL.$$

5. Assuming that the area of a circle is equal to half the product of its circumference and its radius, prove that the area of a circle is a mean proportional between the areas of any two similar polygons, one of which is circumscribed about the circle, the other having the same perimeter as the circle.



6. AB is a diameter of a circle whose radius is 2 inches, and AC is a chord such that the angle BAC is 30°. Find the area and the perimeter of the figure $BACB$, correct to two decimal places.

SEPTEMBER, 1902

(One hour and a half.)

1. Prove that, if straight lines are drawn from a point within a triangle to the extremities of a side, their sum is less than the sum of the other two sides of the triangle.

Prove that the sum of the three straight lines drawn from any point within a triangle to the three vertices is less than the sum of the three sides of the triangle.

2. Prove that, if the sum of two opposite angles of a quadrilateral is two right angles, the quadrilateral can be inscribed in a circle.

Can any other parallelogram than a rectangle be inscribed in a circle? Prove the correctness of your answer.

3. Within a rectangle which is not a square, circles are drawn tangent to two sides of the rectangle. Find the locus of the centres of these circles.

4. Prove that the four bisectors of the angles of a quadrilateral form a second quadrilateral which can be inscribed in a circle.

What exceptions are there to this theorem?

5. Prove that a plane surface may be entirely covered (as in the construction of a pavement) by a combination of regular octagons and of squares whose sides coincide with sides of the octagons.

6. The perpendicular from the vertex of the right angle of a right triangle upon the hypotenuse divides the hypotenuse into segments which are 4 inches and 9 inches long, respectively. Find the area and the perimeter of the triangle.

JUNE, 1903

(One hour and a half.)

1. Prove that if two triangles have a side and two adjacent angles of one respectively equal to a side and two adjacent angles of the other, the triangles are equal.

Prove that if the bisector of an angle of a triangle is perpendicular to the opposite side, the triangle is isosceles.

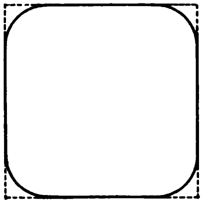
2. Prove that two parallel straight lines intercept equal arcs on a circumference.

Prove that if a trapezoid be inscribed in a circle, the non-parallel sides, produced, will meet on the perpendicular bisector of the parallel sides.

3. Given a closed figure composed of a semicircumference and a diameter. A line of given length moves with one extremity on the perimeter of the given figure, and in such a manner that it always remains parallel to a fixed direction. Assuming that the fixed direction makes an angle of 45° with the diameter, find the locus of the other extremity of the moving line. Describe the locus accurately and prove the truth of your answer.

4. Prove that the radius of the circle inscribed in a right triangle is equal to the semiperimeter of the triangle diminished by the hypotenuse.

5. Draw two lines, AB and CD, such that AB is three times as long as CD. Construct the mean proportional between AB and CD.



6. The corners of a square whose side is 2 inches are rounded off as in the figure by arcs of equal circles, tangent to the sides.

The perimeter of the figure so formed is $7\frac{1}{7}$ inches. Find the common radius of the circles. Compute the area of the figure. Take $\pi = 3\frac{1}{7}$.

JUNE, 1903

(Two hours.)

Omit one starred question.

1. Prove that if two triangles have a side and two adjacent angles of one respectively equal to a side and two adjacent angles of the other, the triangles are equal.

Prove that if the bisector of an angle of a triangle is perpendicular to the opposite side, the triangle is isosceles.

2. Prove that two parallel straight lines intercept equal arcs on a circumference.

Prove that if a trapezoid be inscribed in a circle, the non-parallel sides, produced, will meet on the perpendicular bisector of the parallel sides.

*** 3.** Given a closed figure composed of a semicircumference and a diameter. A line of given length moves with one extremity on the perimeter of the given figure, and in such a manner that it always remains parallel to a fixed direction. Assuming that the fixed direction makes an angle of 45° with the diameter, find the locus of the other extremity of the moving line. Describe the locus accurately, and prove the truth of your answer.

* **4.** Prove that the radius of the circle inscribed in a right triangle is equal to the semiperimeter of the triangle diminished by the hypotenuse.

* **5.** Draw two lines, *AB* and *CD*, such that *AB* is three times as long as *CD*. Construct the mean proportional between *AB* and *CD*.

* **6.** The corners of a square whose side is 2 inches are rounded off as in the figure by means of arcs of equal circles tangent to the sides. The perimeter of the figure so formed is $7\frac{1}{7}$ inches. Find the common radius of the circles. Compute the area of the figure. Take $\pi = 3\frac{1}{7}$.

7. Prove that through any given straight line a plane can be passed perpendicular to any given plane.

Is it true that if two lines are perpendicular to each other, any plane passed through one of the lines is perpendicular to the other? Give the reason for your answer.

8. Prove that the volume of a circular cylinder is equal to the product of its base and altitude.

9. A certain dome is in the form of a spherical zone of one base. The base is a circle 24 feet in diameter, and the highest point is 9 feet above the plane of the base. Find the number of square feet in the surface of the dome correct to two decimal places.

SEPTEMBER, 1903

(One hour and a half.)

1. Prove that when two straight lines are cut by a third, if the alternate interior angles are equal, the two straight lines are parallel.

Prove that when the sides of one angle are perpendicular respectively to the sides of another, the bisector of the first angle is either parallel or perpendicular to the bisector of the second. When are the two bisectors parallel? When perpendicular?

2. Prove that if in a right triangle a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the perpendicular is a mean proportional between the segments of the hypotenuse.

Prove that if the perpendicular from the vertex of a triangle upon the opposite side divides it into two triangles similar to each other, the given triangle is either isosceles or a right triangle.

3. Circles are drawn so that they are always wholly within a given triangle and tangent to two and only two of its sides. What is the locus of the centres of these circles? Prove the correctness of your answer.

4. Prove that in any triangle the product of two sides is equal to the product of the diameter of the circumscribed circle by the perpendicular let fall upon the third side from the vertex of the opposite angle.

5. Prove that if six squares be constructed on the sides of a regular hexagon, and outside of the hexagon, the twelve exterior vertices of these squares are the vertices of a regular dodecagon.

6. Find the area of an equilateral triangle which is inscribed in a circle whose radius is 10 inches.

JUNE, 1904

(One hour and a half.)

1. Prove that if two right triangles have the hypotenuse and a side of one respectively equal to the hypotenuse and a side of the other, the triangles are equal.

Draw a square *ABCD*. On the diagonal *AC* take the point E so that $AE = AB$, and draw through E a line perpendicular to AE, cutting BC in F. Prove that

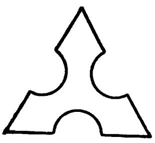
$$BF = FE = EC.$$

2. Prove that an angle formed by two chords of a circle intersecting each other within the circumference is measured by one-half the sum of the arcs intercepted by its sides and by the sides of its vertical angle.

Take five points A, B, C, D, E on the circumference of a circle in the order given. Let the middle point of the arc ABC be F, and the middle point of the arc AED be G. Let the chord FG cut the chord AC in H and the chord AD in K. Prove that AH is equal to AK.

NOTE. The points B and E are used only to indicate the arcs in which the points F and G lie.

3. Given a square with the side 3 inches long. Find the locus of a point P such that the distance from P to the nearest point of the perimeter of the square is 1 inch. Describe the locus accurately.



4. Semicircles are drawn with their centres at the middle points of the sides of an equilateral triangle, forming a figure as here shown.

Prove that if the perimeter of this figure is one-fifth greater than that of the triangle, its area is about one-third less than that of the triangle. If the side of the triangle is 10 inches, what is the area of the figure, correct to one per cent?

5. On the sides of an equilateral triangle ABC as bases equal isosceles triangles ABP, ACQ, BCR, are constructed; the first two are exterior to the given triangle, while R is on the same side of BC as A. Prove that $APRQ$ is a rhombus.

SEPTEMBER, 1904

(One hour and a half.)

1. Prove that if two triangles have three sides of one respectively equal to three sides of the other, the triangles are equal.

Take two points *A* and *B* on the line *OL*. On the line *OM* take points *C* and *D* so that *OC* is equal to *OA* and *OD* is equal to *OB*; draw *BC* and *AD*, intersecting in *E*. Prove that *E* is in the bisector of the angle *LOM*.

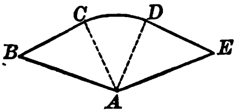
2. Prove that an angle inscribed in a circumference is measured by one-half its intercepted arc.

In a quadrilateral *ABCD* the diagonals *AC* and *BD* are drawn. It is found that the angle *DAC* is equal to the angle *DBC*. Prove that the angles *DAB* and *DCB* are supplementary.

3. What is the locus of points equidistant from two given intersecting straight lines, each line extending indefinitely in both directions? Prove the truth of your statement. State without proof how the answer to the first part of the question must be modified if the given lines are parallel.

4. A figure is made up of a sector of a circle and two equal isosceles right triangles as here shown.

The angles *BAC* and *DAE* are each 45°; the angle *CAD* is 33° 20′. The length of the arc *CD* is 110 feet. Compute the area of the figure correct to one per cent. Take $\pi = \frac{22}{7}$.



5. Prove that in a regular pentagon two diagonals, not drawn from a common vertex, divide one another in extreme and mean ratio.

Suggestion. Let the diagonals *AC* and *BD* intersect in *F*. Compare the triangles *ABC* and *BCF*.

JUNE, 1905

(One hour and a half.)

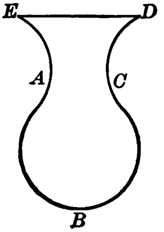
1. Prove that if two triangles have two sides and the included angle of one respectively equal to two sides and the included angle of the other, the triangles are equal.

Let *ABC* be an equilateral triangle, and on the sides *AB*, *BC*, *CA* lay off *AD*, *BE*, *CF*, each equal to $\frac{1}{3}$ *AB*, and join the points *D*, *E*, *F* with one another. Prove that the triangle *DEF* is equilateral, and that its sides are respectively perpendicular to the sides of the given triangle.

2. Prove that when two tangents to the same circle intersect each other, the distances from their point of intersection to their points of contact are equal.

On the circumference of a circle take two points subtending a right angle at the centre, and a third point on the arc between these two. Prove that the perimeter of the triangle formed by the tangents at these three points is equal to the diameter of the circle.

3. An indefinite straight line moves in such a way that it always passes through at least one vertex of a given square, but never crosses the square. What is the locus of the foot of the perpendicular dropped on the moving line from the centre of the square? Describe the locus accurately, and prove the correctness of your answer.



4. The figure here shown is made up of the arc *ABC*, which is three-quarters of a circle; two arcs, *AE* and *CD*, each a quarter of a circle of the same radius and tangent to the first arc at *A* and *C* respectively; and the straight line *DE*. Find the area and the perimeter of the figure if the radius of the circles is a. In the special case $a=3$, compute the area within one-tenth of one per cent of its true value.

5. Circumscribe a circle about the triangle ABC, and let M be the centre of the inscribed circle. Draw the line AM and prolong it to intersect the circumscribed circle at D. Prove that the straight lines BD, DC, and DM are equal.

SEPTEMBER, 1905

(One hour and a half.)

1. Prove that two straight lines perpendicular to the same straight line are parallel.

The diagonals of a quadrilateral divide it into four triangles. Prove that the centres of the circles circumscribed about these four triangles lie at the vertices of a parallelogram. When will this parallelogram be a rectangle?

2. Prove that if two triangles have an angle of one equal to an angle of the other, and the sides including these angles proportional, the triangles are similar.

Prove that if two diagonals of a quadrilateral divide each other in the same ratio, the quadrilateral is a trapezoid.

3. A straight line of fixed length moves with its extremities on two indefinite perpendicular straight lines in a plane. Find the locus of the middle point of the moving line, and prove the correctness of your answer.

A straight line 3 inches long moves with its extremities on the perimeter of a square whose side is 4 inches long. Describe accurately the locus of the middle point of the moving line, giving the dimensions of the resulting figure.

4. A six-pointed star is formed by producing the sides of a regular hexagon until they meet. Find the area of the star, the side of the hexagon being a. If the distance between two opposite vertices of the star is 10 inches, compute the area correct to three significant figures.

5. Draw a straight line from the vertex A of a parallelogram $ABCD$, cutting the side BC in E, and the diagonal BD in F. If $BE=\frac{BC}{n}$, prove that $BF=\frac{BD}{n+1}$.

JUNE, 1906

(One hour and a half.)

The University provides a Syllabus.

1. Prove that the sum of the three angles of any triangle is equal to two right angles.

On a line AE choose a point B, and construct an isosceles triangle ABC with AB as base, the base angles being less than 45°. With B as vertex construct an isosceles triangle BCD whose base CD lies in AC produced. Show that the angle DBE is three times the angle A.

2. Prove that the bisector of an angle of a triangle divides the opposite side into segments proportional to the sides of the angle.

The hypotenuse of a right triangle is 10 inches long and one of the acute angles is 30°. Compute the lengths of the segments into which the short side is divided by the bisector of the opposite angle.

3. A chord BC of a given circle is drawn, and a point A moves on the longer arc BC. Draw the triangle ABC, and find the locus of the centre of a circle inscribed in this triangle.

4. Three equal circular plates are so placed that each touches the other two, and a string is tied tightly around them. If the length of the string is 10 feet, find the radius of the circles correct to three significant figures.

5. Let O be the centre of a circle and P any point outside. With P as centre and radius PO draw the arc of a circle, cutting the given circle at A and B. With A and B as centres and AO as radius draw the arcs intersecting in Q. Prove:

(*a*) that the points O, Q, P are in a straight line; and
(*b*) that $OP \times OQ = (OA)^2$.

Suggestion. Join A and B with O, P, and Q, and join Q with O and P.

JUNE, 1906

(Two hours.)

The University provides a Syllabus.

Omit one starred question.

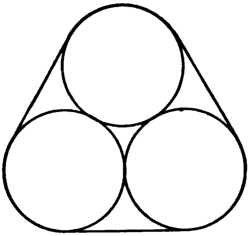
1. Prove that the sum of the three angles of any triangle is equal to two right angles.

On a line AE choose a point B, and construct an isosceles triangle ABC with AB as base, the base angles being less than 45°. With B as vertex construct an isosceles triangle BCD whose base CD lies in AC produced. Show that the angle DBE is three times the angle A.

2. Prove that the bisector of an angle of a triangle divides the opposite side into segments proportional to the sides of the angle.

The hypotenuse of a right triangle is 10 inches long, and one of the acute angles is 30°. Compute the lengths of the segments into which the short side is divided by the bisector of the opposite angle.

* **3.** A chord BC of a given circle is drawn, and a point A moves on the longer arc BC. Draw the triangle ABC, and find the locus of the centre of a circle inscribed in this triangle.



* **4.** Three equal circular plates are so placed that each touches the other two, and a string is tied tightly around them. If the length of the string is 10 feet, find the radius of the circles correct to three significant figures.

* **5.** Let O be the centre of a circle and P any point outside. With P as centre and radius PO draw the arc of a circle, cutting the given circle at A and B. With A and B as centres and AO as radius draw arcs intersecting in Q. Prove:

(*a*) that the points O, Q, P are in a straight line; and
(*b*) that $OP \times OQ = (OA)^2$.

Suggestion. Join A and B with O, P, and Q, and join Q with O and P.

6. Prove that if a straight line is perpendicular to a plane, every plane passed through the line is perpendicular to the plane.

If two straight lines lie in the same plane and are perpendicular to a third straight line, are they parallel? Give your reasons.

7. Prove that two triangles on the same sphere are either equal or symmetrical when the three sides of one are respectively equal to the three sides of the other.

8. Find the volume of a spherical segment of one base whose altitude is one-half the radius of the sphere. (Let the radius of the sphere be R.)

SEPTEMBER, 1906

(One hour and a half.)

The University provides a Syllabus.

1. Prove that if the pairs of opposite sides of a quadrilateral are equal, the figure is a parallelogram.

In a quadrilateral which has no two parallel sides, prove that the middle points of a pair of opposite sides and the middle points of the two diagonals are the vertices of a parallelogram.

2. Prove that the angle formed by a tangent and a chord is measured by one-half the intercepted arc.

Two circles intersect at A and B. The two tangents at A are drawn, cutting the circles in C and D respectively. Prove that

$$(AB)^2 = BC \times BD.$$

Suggestion. Prove the triangles ABC and DBA similar.

3. In the rectangle $ABCD$ the side AB is twice as long as the side BC. A point P is taken on the side AB, and a circle is drawn through the points CDP. Describe accurately the path of the centre of this circle when P moves from A to B.

4. A square of side $2a$ is turned about its centre through an angle of 45°. Compute the perimeter of the eight-pointed star formed by the two squares.

5. Within a given circle two circles of half the diameter are drawn tangent to the large circle at any two points A and B. Prove that the straight line AB always passes through a point of intersection of the two small circles.

JUNE, 1907

(One hour and a half.)

The University provides a Syllabus.

1. Show that if two parallel straight lines are cut by a third straight line, the alternate interior angles are equal.

Prove the correctness of the following construction for bisecting an angle ABC: Upon AB produced beyond B take BD equal to BC, and draw a line through B parallel to DC.

2. Prove that the diameter of a circle perpendicular to a given chord bisects that chord.

Show how to construct a chord through a given point A within a circle so that the extremities of the chord shall be equidistant from another given point B.

3. A rod of length a is free to move within a semicircular area of radius a. Describe accurately the boundary of the region within which the middle point of the rod will always be found.

4. A roadway 60 feet wide is cut through the middle of a circular field 120 feet in diameter. Compute the area of the remainder of the field, correct to one per cent of its value.

5. A certain instrument known as Peaucellier's " Inversor " is constructed as follows: Two equal rods OA and OB, each of length a, are jointed together at O; four shorter rods, each of length b, form the sides of a quadrilateral jointed at the vertices A, P, B, Q. Show that P and Q are in line with O, and that

$$OP \times OQ = a^2 - b^2.$$

SEPTEMBER, 1907

(One hour and a half.)

The University provides a Syllabus.

1. Prove that in an isoceles triangle the angles opposite the equal sides are equal.

On BC, the longest side of a triangle ABC, points B' and C' are taken so that $BB'=BA$ and $CC'=CA$. Show that the angle $B'AC'$ equals half the sum of the angles B and C.

2. Prove that the area of a triangle is one-half the product of the base and the altitude.

Show that if a point move about within a regular polygon, the sum of the perpendiculars let fall upon the sides (or the sides produced) will be constant.

3. A rod 8 feet long is free to move within a rectangle 8 feet long and 6 feet wide. Describe accurately the boundary of the region within which the middle point of the rod will be always found.

4. A circle is described upon one side of an equilateral triangle as diameter. Compute the area of the part of the triangle which lies outside the circle, correct to one per cent of its value.

5. Two circles intersect at right angles. The radius of one of them is of length a, and its centre is the point O. Show that if any line be drawn through O cutting the second circle in the points P and P', then

$$OP\times OP'=a^2.$$

JUNE, 1908

(One hour and a half.)

The University provides a Syllabus.

1. Prove that if in a quadrilateral a pair of opposite sides be equal and parallel, the figure is a parallelogram.

In a certain quadrilateral, one diagonal and a line connecting the middle points of a pair of opposite sides bisect each other. Prove that the quadrilateral is a parallelogram.

2. Prove that in any circle equal chords subtend equal arcs.

Show that the bisector of an angle of a triangle meets the perpendicular bisector of the opposite side on the circumference of the circumscribed circle.

3. The radii of two circles are 1 inch, and $\sqrt{3}$ inches respectively, and the distance between their centres is 2 inches. Compute their common area to three significant figures.

4. Determine a point P without a given circle so that the sum of the lengths of the tangents from P to the circle shall be equal to the distance from P to the farthest point of the circle.

5. The image of a point in a mirror is, apparently, as far behind the mirror as the point itself is in front. If a mirror revolve about a vertical axis, what will be the locus of the apparent image of a fixed point 1 foot from the axis?

SEPTEMBER, 1908

(One hour and a half.)

The University provides a Syllabus.

1. Prove that the perpendicular bisector of the base of an isosceles triangle bisects the opposite angle.

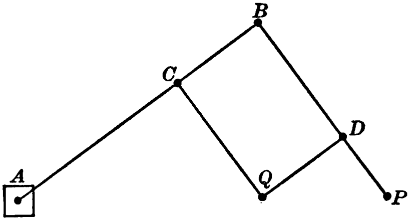
Assuming that a billiard ball rebounds from a side or cushion of the table at the same angle at which it strikes, find the point at which a player should aim his ball if he wishes to strike another ball after one rebound. Prove the correctness of your construction.

2. Prove that, if two triangles have an angle of one equal to an angle of the other and the including sides proportional, the triangles are similar.

Show that the middle points of the diagonals, and of the non-parallel sides of a trapezoid lie in line.

3. Compute to three significant figures the length of the shortest piece of wire which will just go around two pipes lying side by side, the diameters being 2 inches and 6 inches respectively.

4. Four rods AB, BP, CQ, QD are pivoted together so that $BC = DQ$, $BD = CQ$. The point A is fixed. Show that if AQP are once in line, they will remain in line when P moves freely about the plane, and show that AQ will bear a constant ratio to AP. (This instrument is called a pantograph.)



5. A point P lies within a fixed circle. Find the locus of points which lie one-third of the way from P to the points of the circumference.

JUNE, 1909

(Two hours.)

The University provides a Syllabus.

1. Show that if a triangle be equilateral, it is also equiangular.

Is this theorem true in the case of a quadrilateral? Give your reason.

2. Prove that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Deduce from this a proof that of two oblique lines drawn from the same point of a perpendicular and cutting off unequal distances from the foot of the perpendicular, the more remote is the greater.

3. Prove that if the products of the segments into which the diagonals of a quadrilateral divide one another are equal, a circle may be circumscribed about the quadrilateral.

4. A square 10 inches on a side is changed into a regular octagon by cutting off the corners. Find the area of this octagon.

5. A wheel 40 inches in diameter has a flat place 5 inches long on the rim. Describe carefully the locus of the centre as the wheel rolls along the level.

SEPTEMBER, 1909

(Two hours.)

The University provides a Syllabus.

Take $\pi = \frac{22}{7}$.

1. Prove that the opposite sides and the opposite angles of a parallelogram are equal.

Prove that the only sort of a parallelogram that can be inscribed in a circle is a rectangle.

2. Show, with proof, how to inscribe a circle in a given triangle. Explain the difference between this problem and that of constructing a circle to touch three given lines.

3. Show that the areas of two similar triangles are to each other as the squares of any two homologous sides.

Show that the quadrilateral formed by joining the middle points of the adjacent sides of a given quadrilateral is a parallelogram whose area is one-half that of the quadrilateral.

4. At two points A and B of the circumference of a circle whose centre is O, tangents are drawn. The points A' and B' lie respectively on these tangents, and the angle $OA'B'$ is a right angle. Prove that $BB'^2 = AA'^2 + A'B'^2$.

5. A circular basin of radius r is full of water, and upon the surface there floats a thin straight stick of length 14. Find the area of that region of the surface which is inaccessible to the middle point of the stick.

JUNE, 1910

(Two hours.)

The University provides a Syllabus.

1. Prove that if the sides of one angle be perpendicular respectively to those of another, the angles are either equal or supplementary.

2. Show how to inscribe a circle in a given triangle.

How many circles can be drawn to touch three given lines? Are there any positions of these lines for which the number is less?

3. Define "incommensurable magnitudes." Give a careful proof of some theorem where such magnitudes occur.

4. $ABCD$ are the vertices in order of a quadrilateral which is circumscribed to a circle whose centre is O. Prove that $\angle AOB$ and $\angle COD$ are supplementary.

5. Two radii of a circle OA and OB make a right angle. A second circle is described upon AB as diameter. Prove that the area of the crescent-shaped region outside of the first circle, but inside of the second, is equal to that of the triangle AOB. (Hippocrates, fifth century B. C.)

SEPTEMBER, 1910

(Two hours.)

The University provides a Syllabus.

1. Prove that if two equal oblique lines be drawn from a point to a line they must meet it at equal distances from the foot of the perpendicular.

Prove that a straight line cannot intersect a circumference more than twice.

2. Prove that if two circles intersect, the common chord is perpendicular to the line of centres.

3. Prove that in a right triangle the perpendicular on the hypotenuse is a mean proportional between the segments of the hypotenuse.

In a right triangle ABC, whose right angle is at B, P is the foot of the perpendicular from B upon the hypotenuse AC, and BP produced intersects in D the perpendicular to AB at A. Prove that $PA \times PB = PC \times PD$. (Plato, fourth century B. C.)

4. A square is rolled along a straight line without slipping. What is the length of the locus of one corner as the square makes one complete turn?

5. A regular hexagon is reduced to one of smaller size by cutting straight across from the middle point of each side to that of the next. What proportion of the area is removed?

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO YALE

JUNE, 1889

(Time allowed, one hour.)

1. Solve one of the two following problems with ruler and compass. No explanation is to be written, but each step of the construction is to be made clear on the figure.

(*a*) Construct a right-angled isosceles triangle and inscribe a circle in it.

(*b*) Divide a straight line AB into three equal parts. Erect a square on the middle part and construct a triangle of equal area.

2. From two points A and B, on an arc of a circle straight lines, AC, AD, BC, BD, are drawn to the ends of the chord of the arc. Find two similar triangles in the figure, prove them similar, and write the proportions of their homologous sides.

3. Prove that in equal circles two incommensurable arcs have the same ratio as the angles which they subtend at the centre.

4. Prove that two triangles having a common angle are to each other as the products of the sides including the common angle.

5. Solve or prove one of the following propositions:

(*a*) To circumscribe about a circle a regular polygon similar to a given inscribed polygon.

(*b*) The homologous sides of similar regular polygons have the same ratio as the radii of their circumscribed circles, and their perimeter have the same ratio as these radii.

JUNE, 1890

(Time allowed, one hour.)

1. Prove that the bisectors of the angles of a triangle meet in a point.

2. On a given straight line AB construct a segment of a circle containing an angle equal to a given acute angle C.

(Show clearly on the figure the method of construction. Ruler and compass must be used. The proof is not required.)

3. Prove that in any triangle the square on the side opposite an acute angle is equivalent to the sum of the squares of the other two sides diminished by twice the product of one of these sides and the projection of the other upon that side.

4. Prove that regular polygons of the same number of sides are similar figures.

5. If the radius of a circle is 5 inches, compute its circumference and its area; also the perimeter, the area, and the apothem of an inscribed square.

JUNE, 1891

(Time allowed, one hour.)

1. Prove that two straight lines which are both parallel to a third straight line are parallel to each other.

2. Prove that if from a point without a circle there be drawn a line tangent to the circle and also a line cutting it and not passing through the centre, the tangent is a mean proportion between the whole secant and its external segment.

3. Construct an equilateral triangle whose perimeter shall be 12 inches and inscribe in it a circle.

(NOTE. Write nothing in connection with this problem.)

4. Prove that if the points of tangency of the above inscribed circle be joined the triangle formed is equilateral, and its perimeter is 6 inches.

5. (*a*) Define a geometrical locus, and give examples of loci from the figure drawn in number 3.

(*b*) Define the limit of a variable, and write the caption of some proposition whose proof depends on the theory of limits.

JUNE, 1892

(Time allowed, one hour.)

1. Construct accurately by ruler and compass a parallelogram $ABCD$ having the angle A 45°, the side AB 6 units in length and the altitude 3 of the same units.

Calculate the length of AC.

2. (*a*) State the converse of the following proposition:

If a triangle is isosceles and if a straight line is drawn through the vertex parallel to the base, it bisects an exterior angle of the triangle.

(*b*) Prove the converse as you have stated it.

(Make the demonstration as full and clear as possible.)

3. Prove two of the following propositions; the work may be limited to drawing a figure and giving a synopsis of the demonstration:

(*a*) If the area of a regular polygon is equal to the product of the perimeter by one-half the apothem, it follows that the area of a circle $=\pi R^2$.

(*b*) If two lines are drawn through the same point across a circle, the products of the two distances on each line from this point to the circumference are equal to each other.

(*c*) If the radius of a circle be divided in extreme and mean ratio, the greater segment is equal to one side of a regular inscribed decagon.

GEOMETRY (A), JUNE, 1895

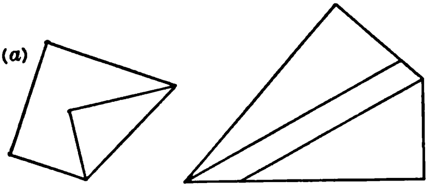
(Time allowed, 60 minutes.)

1. (*a*) Define the terms "locus" and "limit of a variable" and give an example of each.

(*b*) Prove that two triangles are similar if their homologous sides are proportional.

(*c*) Through a given point within a circle draw two equal chords.

[Both the construction (with ruler and compass), and also the proof, are required.]



2 (*a*) Prove that if each of two angles of a quadrilateral is a right angle, the bisectors of the other angles are either perpendicular or parallel to each other.

(*d*) Prove that if the radius of a circle is divided in extreme and mean ratio, the greater part is equal to the side of a regular inscribed decagon.

(The construction is not required.)

GEOMETRY (B), JUNE, 1895

(Time allowed, 45 minutes.)

One question may be omitted. Logarithmic tables should be used in calculating the answers of two questions.

1. The base of a triangle is 14 inches and its altitude is 7 inches. Find the area of the trapezoid cut off by a line 6 inches from the vertex.

Express the result in square meters.

2. Find the number of feet in an arc 40° 12′ if the radius of the circle is 0.7539 meters.

3. The length of a chord is 10 feet, and its greatest distance from the subtending arc is 2 feet $7\frac{1}{2}$ inches. Find the radius of the circle.

4. Find the area, and also the weight in grams, of the largest square that can be cut from a circular sheet of tin 16 inches in diameter and weighing 8.2 ounces per square foot.

PLANE GEOMETRY (A), JUNE, 1896

(Time, one hour.)

1. The sum of three angles of a triangle is equal to two right angles.

2. Construct a circle having its centre in a given line and passing through two given points.

3. The bisector of the angle of a triangle divides the opposite side into segments which are proportional to the two other sides.

4. If two angles of a quadrilateral are bisected by one of its diagonals, the quadrilateral is divided into two equal triangles and the two diagonals of the quadrilateral are perpendicular to each other.

5. The circumference of two circles are to each other as their radii. (Use the method of limits.)

PLANE GEOMETRY (B), JUNE, 1896

(Time, 45 minutes.)

1. A tree casts a shadow 90 feet long, when a vertical rod 6 feet high casts a shadow 4 feet long. How high is the tree?

2. The distance from the centre of a circle to a chord 10 inches long is 12 inches. Find the distance from the centre to a chord 24 inches long.

3. The diameter of a circular grass plot is 28 feet. Find the diameter of a grass plot just twice as large (use logarithms).

4. Find the area of a triangle whose sides are $a = 12.342$ meters, $b = 31.456$ meters, $c = 24.756$ meters, using the formula, area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{(a+b+c)}{2}$. (Use logarithms.)

JUNE, 1899

1. Two triangles are similar if their homologous sides are proportional.

2. Define the limit of a variable. Prove that if two variables are always equal their limits are equal. Prove that the area of a circle is equal to one-half the product of its radius and circumference.

3. (*a*) In any quadrilateral, if a line be drawn through the middle points of two adjacent sides, and a second line through the middle points of the other two sides, these lines will be parallel.

(*b*) If the middle points of the opposite sides of a quadrilateral be joined, the lines so drawn will bisect each other.

4. (*a*) If two circles are tangent internally and if the radius of the one be the diameter of the other, a chord of the larger drawn through the point of tangency is bisected by the smaller.

(*b*) What example of a locus is found in the previous figure?

JUNE, 1900

(Time, one hour.)

In question 1 the work will not be accepted unless the constructions are made accurately with ruler and compass.

1. Divide a line externally in extreme and mean ratio and prove the construction.

What use is made of this construction in Geometry?

2. (*a*) From a point A on the circumference of a circle two equal chords, AB and AC, are drawn. Prove that they make equal angles with the diameter through A.

(*b*) If each of these angles is 30°, prove that the points A, B and C trisect the circumference.

3. In two similar triangles the bases and altitudes are proportional and in two equivalent triangles the bases and altitudes are inversely proportional.

4. (*a*) Prove that the ratio of the circumference of a circle to its diameter is the same for all circles. What is the approximate value of this ratio?

(*b*) Explain briefly (without proofs) the method of determining this value.

GEOMETRY (A), JUNE, 1901

1. State and prove a theorem relating to the squares of the sides of an obtuse-angled triangle.

2. Construct accurately with ruler and compass two tangents to a circle which shall include between their points of contact an arc of 120° of the circumference, and write the value of the angle included between them.

The proof may be omitted if the method is made clear by the construction of the figure.

3. Define locus of a point.

Prove that the bisector of an angle A of a triangle and the bisectors of its exterior angles B and C meet in a point.

4. An isosceles triangle with vertex A is inscribed in a circle, and through A a line is drawn cutting the side BC at E and the circle at D. Prove that AB is a mean proportional between AD and AE.

5. Quote a theorem, a definition and an axiom used in proving that the area of a triangle is equal to one-half the product of its base and altitude. *Write nothing else.*

GEOMETRY (A), JUNE, 1902

1. What is an axiom? State four axioms. Prove a proposition relating to circles in which one of these axioms is used.

2. Show (without using the area of a triangle) that any two altitudes of a triangle are inversely proportional to the sides upon which they fall.

3. Show how to construct $\sqrt{2}$, $\sqrt{3}$ and the square root of any positive integer.

4. What is meant by the length of the circumference of a circle? Describe how the length is computed in terms of the radius.

5. Show that if two circles are tangent externally the common interior tangent bisects the two common exterior tangents.

GEOMETRY (A), JUNE, 1903

1. If a straight line divides two sides of a triangle proportionally, it is parallel to the base.

2. What is the locus of points equidistant from the sides of an angle? Prove your statement.

3. The areas of two similar triangles are to each other as the squares on two homologous sides.

4. The area of the regular inscribed hexagon is equal to twice the area of the inscribed equilateral triangle and half the area of the circumscribed equilateral triangle.

5. Construct a circle passing through two given points and tangent to a given straight line.

GEOMETRY (A), JUNE, 1904

1. If the opposite sides of a quadrilateral are equal the figure is a parallelogram.

2. An angle included by a tangent and a chord drawn from the point of contact is measured by half the intercepted arc.

3. Construct a circle tangent to two given intersecting straight lines so that it is tangent to one of these lines at a given point.

4. A circle described on one of the equal sides of an isosceles triangle as a diameter cuts the base at its middle point.

5. Assuming the principle that the tangent is the mean proportional between the whole secant and its external segment, prove that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

GEOMETRY (A), JUNE, 1905

(Time allowed, one hour.)

1. Prove that two right triangles having a leg and the adjacent acute angle of one equal to a leg and the adjacent acute angle of the other are equal.

2. Prove by the method of limits that the ratio of the areas of two circles is equal to the ratio of the squares of their radii.

3. Construct with ruler and compass a circle that shall pass through a given point P, and be tangent to a given circle at a fixed point N.

4. Prove that if a tangent and a chord be drawn from the same point on a circumference, the distances from the middle point of the included arc to the tangent and chord are equal.

5. Prove that two chords of a circle drawn from a common point A on the circumference and cut by a line parallel to a tangent through A are divided proportionately.

JUNE, 1906

(Time allowed, an hour and three-quarters.)

(*Questions* 5, 6, *and* 7 *should be answered by candidates removing conditions in Geometry B.*)

1. State and prove the theorem for the area of a trapezoid.

2. The perpendicular bisectors of the sides of a triangle meet in a point.

3. The bisector of any angle of a triangle divides the opposite side into segments proportional to the other two sides.

4. The bisectors of the angles of a quadrilateral circumscribed about a circle meet at a point.

5. Two parallel chords of a circle are of lengths 6 and 8 and are 1 inch apart. Find the radius of the circle.

6. A circle of radius 6 has half its area removed by cutting out a concentric circle. Find the width of the ring that remains.

7. Express the area of a regular hexagon in terms of a when a is one of its sides.

8. Through a given point outside a circle construct a secant such that the intercepted chord shall have the given length (less than the diameter) a. Prove the construction correct.

9. A circle is described on one of the legs of a right triangle as diameter; prove that the tangent at the point where it cuts the hypotenuse bisects the other leg.

JUNE, 1907

(Time allowed, two hours.)

1. If any number of parallels intercept equal segments on one cutting line, they intercept equal segments on every other cutting line.

2. Upon a given line construct accurately with ruler and compasses a segment of circle that includes an angle of 30°.

3. If two polygons are similar to the same polygon, they are similar to each other.

4. If the side of a regular polygon inscribed in a circle of unit radius is a, find the side of the regular inscribed polygon of twice as many sides.

5. Given two parallel lines and a transversal. What angles do the bisectors of the interior angles on the same side of the transversal make with each other? Prove your statement.

6. In a circle two chords (not both of them diameters) cut one another. Prove that the point of intersection does not bisect both chords.

7. Find the area that remains after the inscribed circle is removed from an equilateral triangle, a side of which is 6 inches.

8. The equal sides of an isosceles trapezoid are 5 feet.

The parallel sides are 12 and 18 feet respectively. Find the area.

9. A circle is tangent to a given circle and to a given straight line. Prove that the two points of tangency and the extremity of the diameter of the given circle that is perpendicular to the given line lie in a straight line.

JUNE, 1908

(Time allowed, two hours.)

1. The sum of the interior angles of a polygon of n sides is equal to $(2n-4)$ right angles.

2. Two triangles whose sides are respectively perpendicular to each other are similar.

3. The areas of two similar polygons are to each other as the squares of any two homologous sides.

4. Construct a regular octagon having given one side.

5. If in any triangle a medium be drawn from the vertex to the base, the sum of the squares of the other two sides is equal to twice the sum of the squares of the half-base and the median.

6. Construct a triangle having given the middle points of its three sides.

7. The legs of a right triangle are 6 and 8 respectively. Find their projections on the hypotenuse.

8. Compute the area of the square inscribed in a circle whose perimeter is 63 feet.

9. AB is a line 20 inches long and C is its middle point. On AB, AC, and CB semicircles are constructed. Compute the radius of the circle inscribed in the space inclosed by the three semicircles.

JULY, 1909

(Time allowed, two hours.)

1. If from a point outside a circle a tangent and a secant be drawn, the tangent is a mean proportional between the whole secant and the external segment.

2. What is the locus of points equidistant from the sides of an angle? Prove your statement.

3. The perimeters of two regular polygons of the same number of sides are in the same ratio as their radii, or as their apothems.

What theorem about circles would you infer from this theorem about polygons? Why?

4. In a given straight line find a point such that lines drawn from it to two given points are perpendicular to each other.

5. Construct a square whose area is equal to that of a given equilateral triangle.

6. In the triangle ABC, $AB=8$, $AC=6$, $BC=7$. If the bisectors of the angles at A meet the line BC at the points D and E, find DB and EB.

7. An isosceles trapezoid has parallel sides of lengths 12 and 16, and equal sides of length 6. Can this trapezoid be inscribed in a circle whose diameter is the longest side of the trapezoid?

8. Consider the earth a perfect sphere, and a circular hoop made whose circumference is 10 feet longer than the equator of the earth. If this hoop were placed round the equator with its centre at the centre of the earth, what would be the width of the ring between them?

9. Through a fixed point on the circumference of a circle construct a chord that shall be bisected by a given fixed chord.

JUNE, 1910

1. Two right triangles are equal if the hypotenuse and a leg of one are equal respectively to the hypotenuse and a leg of the other.

2. Upon a given straight line construct a segment of a circle which shall contain a given obtuse angle. Prove your construction correct.

3. If three or more non-parallel lines intercept proportional segments on two parallel lines, they pass through a point.

4. If two lines AB and CD cut at E so that $AE \times EB = CE \times ED$, prove that a circumference can be passed through A, B, C, and D.

5. Find the side of an equilateral triangle equivalent to a circle whose radius is 16.

6. Prove that the locus of the extremities of all the equal tangents that can be drawn to a given circle is a circle concentric to the given circle.

7. If the area of a rhombus is a and one diagonal is d, find the length of a side.

8. If a regular hexagon is inscribed in a given circle and another is circumscribed about the same circle, find the ratio of the areas of the hexagons.

9. If two circles are tangent internally, the chords of the greater circle drawn from the point of contact are divided into proportional segments by the smaller circle.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO SHEFFIELD SCIENTIFIC SCHOOL

JUNE, 1895

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. If two sides of a triangle are unequal, the angles opposite are unequal, and the greater angle is opposite the greater side. Also state and prove the converse proposition.

2. Define a locus. Find the locus of the centres of all circles whose circumferences pass through two given points.

3. An angle formed by two tangents intersecting without the circumference is measured by one-half the difference of the intercepted arcs. Give values of the arcs when the tangents are perpendicular to each other.

4. Construct a circle which shall be tangent to a given line and touch a given circle at a given point.

5. A circle may be circumscribed about any regular polygon; and a circle may also be inscribed in it.

SEPTEMBER, 1895

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. Define an angle, equal angles, oblique angles, complementary angles, supplementary angles, an inscribed angle, an angle inscribed in a segment, a re-entrant angle, the unit of angle.

2. The sum of two perpendiculars dropped from any point in the base of an isosceles triangle to the equal sides is constant and equal to the perpendicular let fall from the vertex of one of the equal angles to the opposite side.

3. The area of a trapezoid is equal to the product of its altitude by half the sum of its parallel sides.

4. Construct a square equivalent to a given parallelogram.

5. Two regular polygons of the same number of sides are similar. If S and S' are their areas, what is the ratio of the circumferences of their circumscribed circles.

JUNE, 1896

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. Two angles whose sides are parallel each to each are either equal or supplementary. When will they be equal, and when supplementary?

2. An angle formed by two chords intersecting within the circumference of a circle is measured by one-half the sum of the intercepted arcs.

3. A triangle having a base of 8 inches is cut by a line parallel to the base and 6 inches from it. If the base of the smaller triangle thus formed is 5 inches, find the area of the larger triangle.

4. Construct a parallelogram equivalent to a given square, having given the sum of its base and altitude. Give proof.

5. What are regular polygons? A circle may be circumscribed about, and a circle may be inscribed in, any regular polygon.

SEPTEMBER, 1896

(NOTE. *State at the head of your paper what text-book you have studied and to what extent.*)

1. Define homologous points, homologous lines, homologous angles, equal figures, equivalent figures, symmetrical figures, isoperimetric figures.

2. Deduce expressions for the sum of the interior angles and the sum of the exterior angles of a polygon of n sides.

3. If through each of the vertices of a given triangle a line be drawn parallel to the opposite side, a new triangle will be formed equal to four times the given triangle.

4. The square described on the hypotenuse of a right triangle is equivalent to the sum of the squares described on the other two sides.

5. Of all triangles having the same base and equal perimeters, the isosceles triangle is the maximum.

JULY, 1897

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. When two parallel lines are cut by a third straight line, the alternate interior angles are equal, and conversely.

2. What is meant by measuring a magnitude? When are two magnitudes of the same kind commensurable and when incommensurable? Explain what is meant by the limit of a varying magnitude, and give illustrations drawn from geometry.

3. The product of two sides of a triangle is equal to the product of the perpendicular on the third side let fall from their intersection and the diameter of the circumscribed circle.

4. Show how to construct a square equivalent to the sum of two given squares.

5. Show how to inscribe a regular hexagon in a circle of radius R, and prove that the area of the hexagon is equal to $\frac{3}{2}R^2\sqrt{3}$.

SEPTEMBER, 1897

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. Enumerate and define the different kinds of quadrilaterals. Show that the diagonals of a rhombus are perpendicular.

2. Upon a given line to describe a segment of a circle to contain a given angle.

3. What is meant by the locus of a point? If through a point within a circle a chord of the circle is drawn, find the locus of the middle point of that chord.

4. (*a*) The perimeters of regular polygons of the same number of sides have the same ratio as any two homologous sides.

(*b*) The areas of regular polygons of the same number of sides have the same ratio as the squares of any two homologous sides.

5. Find the ratio of the areas of a square and the inscribed circle. Is this ratio commensurable?

JUNE, 1898

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. (*a*) Two lines perpendicular to the same line are parallel.

(*b*) If two lines are parallel, and one of them is perpendicular to a third line, then the second line is also perpendicular to the third line.

2. (*a*) Of two unequal chords in a circle, the lesser chord is farther from the centre.

(*b*) Of two chords of a circle unequally distant from the centre, the nearer is the greater.

3. Define similar polygons, similar sectors, similar segments. If two polygons are composed of the same number of triangles, similar each to each and similarly placed, the polygons are similar.

4. The areas of similar segments have the same ratio as the squares of the radii.

5. Show that the apothem of an equilateral triangle inscribed in a circle is equal to one-half the radius of the circle.

SEPTEMBER, 1898

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. Each vertex of a triangle is joined by a straight line to the middle point of the opposite side; show that these lines meet in a point.

2. The bisector of an interior or exterior angle of a triangle divides the opposite side into segments proportional to the other two sides.

3. The area of a trapezoid equals the altitude times the line joining the middle points of the non-parallel sides.

4. (*a*) The circumferences of two circles have the same ratio as the radii.

(*b*) The areas of two circles have the same ratio as the squares of the radii.

5. The area included between the circumferences of two circles having the same centre equals the area of a circle whose radius is the length of a tangent to the smaller circle drawn from any point on the circumference of the larger circle.

1899

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. (*a*) The sum of the angles of a triangle equals two right angles.

(*b*) State and prove the theorem on the sum of the angles of any polygon.

2. If two circumferences intersect, the straight line joining their centres bisects their common chord at right angles.

State the corresponding theorem when the circumferences are *tangent* to each other.

3. A straight line parallel to one side of a triangle divides the other two sides proportionally.

4. When is a straight line divided in *extreme and mean ratio?*

Give and prove the construction for dividing a given straight line in extreme and mean ratio.

To the construction of what regular polygon does this construction apply?

5. What is the meaning of π in Geometry?

Find the length of the side of an equilateral triangle whose area equals that of a circle of radius R.

1900

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. The three perpendiculars erected at the middle points of the sides of a triangle meet in a common point.

2. State and prove the theorems regarding the measurement of the angle between (*a*) two chords of a circle; (*b*) two secants of a circle.

3. When are two magnitudes *commensurable?* When *incommensurable?*

Prove that the areas of two rectangles with equal bases are in the same ratio as the altitudes, both when the latter are commensurable and incommensurable.

4. The areas of two triangles having an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles.

5. Given a square the length of whose side is 6 units, construct a rectangle with altitude 2 units and equivalent to the square.

6. What is the meaning of π in Geometry? State and prove the theorem on the area of a circle.

SEPTEMBER, 1900

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. What is a trapezoid? A rhombus?

Prove that the diagonals of a rhombus bisect each other at right angles.

2. To draw a tangent to a circle from an external point.

3. When is a variable magnitude said to have a limit? Give geometrical examples illustrating the definition.

4. What are similar polygons?

Two triangles whose homologous sides are proportional are similar.

5. The product of two sides of any triangle equals the product of the diameter of the circumscribed circle and the altitude upon the third side.

6. One of the equal sides of an isosceles triangle is a and the angle at the vertex is 30°; show that the length of the base equals

$$a\sqrt{2-\sqrt{3}}.$$

7. The areas of two regular polygons of the same number of sides are in the same ratio as the squares of the radii of the inscribed or circumscribed circles.

JUNE, 1901

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. Of two oblique lines drawn from the same point in a perpendicular and cutting off unequal distances from the foot, the more remote is the greater.

State and prove the converse theorem.

2. What is meant by measuring a magnitude? Define area of any plane figure.

If the side of a given square be taken as the linear unit show that the diagonal of that square is incommensurable.

3. Tangents are drawn at the extremities of a chord of a circle to meet in a point P; then the radius of the circle is a mean proportional between the distances of the chord and of P from the centre.

4. To construct a triangle equivalent to a given polygon.

5. The perimeters of two similar polygons have same ratio as any two homologous sides.

6. The radius of a circle is 10 inches; find the area of a segment whose arc is 60°.

SEPTEMBER, 1901

(NOTE. *State at the head of your paper what text-book you have studied on the subject and to what extent.*)

1. If two triangles have two sides of one equal respectively to two sides of the other, but the included angle of the first greater than the included angle of the second, then the third side of the first is greater than the third side of the second. State and prove the converse theorem.

2. To divide a given straight line into segments proportional to given straight lines.

3. In any triangle the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides, minus twice the product of one of these sides and the projection of the other side upon it.

4. State and prove the theorem for the area of a regular polygon.

5. If from any point within a regular polygon of n sides perpendiculars are drawn to the several sides, the sum of these perpendiculars is equal to n times the apothem.

6. Of all triangles having equal perimeters and one common side the isosceles triangle is the greatest in area.

JUNE, 1902

1. What text-book have you used and how long have you studied this subject?

2. Give and prove the construction for the circumscribed and inscribed circles of a triangle.

3. Show that the square constructed upon the hypotenuse of an isosceles right triangle is equal to four times that triangle.

4. The bisector of an angle of a triangle divides the opposite side into segments which are proportional to the other two sides. State and prove the corresponding theorem for the bisector of an exterior angle of a triangle.

5. Find the locus of the middle point of a line of given length whose extremities move along two perpendicular lines.

6. Two rectangles having equal bases are to each other as their altitudes, both when the latter are commensurable and incommensurable.

7. To construct a circle equivalent to the sum of two given circles.

8. If the area of an equilateral triangle is 164.51 square inches, find its perimeter.

SEPTEMBER, 1902

1. What text-book have you used and how long have you studied this subject?

2. If any number of parallels intercept equal parts on one cutting line, they intercept equal parts on every other cutting line.

3. The triangle formed by joining the middle point of one of the non-parallel sides of a trapezoid to the extremities of the opposite side is equivalent to one-half the trapezoid.

4. Enumerate the various cases of similar triangles and prove any one of them.

5. Through a fixed point A on a circle let a chord AB be drawn. If this chord be produced through A to a point P, such that the ratio $AP:AB$ is constant, find the locus of P.

6. In the same circle or in equal circles two angles at the centre have the same ratio as their intercepted arcs.

7. Construct an equilateral triangle which shall be equivalent to a given parallelogram.

8. From a point without a circle two tangents are drawn making with each other an angle of 60°. The length of each tangent is 15 inches. Find the diameter of the circle.

JUNE, 1903

1. What text-book have you used, and how long have you studied this subject?

2. In the same circle or equal circles, the less of two chords is at the greater distance from the centre; conversely, the chord at the greater distance from the centre is the less.

3. A right triangle may be divided into two isosceles triangles.

4. State and prove the theorems which hold for the figure formed by dropping a perpendicular from the vertex of a right triangle upon the hypotenuse.

5. If two tangents are drawn to a circle at the extremities of a diameter, the portion of any third tangent intercepted between them is divided at its point of contact into segments whose product equals the square of the radius.

6. Prove the theorems concerning the length of circumference and the area of a circle.

7. Find the area of the segment subtended by the side of the regular hexagon inscribed in a circle of radius R.

SEPTEMBER, 1903

1. What text-book have you used, and how long have you studied this subject?

2. Two angles whose sides are perpendicular each to each are either equal or supplementary.

3. If L and M are the middle points of the sides AB, CD of a parallelogram $ABCD$, the straight lines DL, BM trisect the diagonal AC.

4. The areas of two triangles which have an angle of one equal to an angle of the other are to each other as the products of the sides including those angles.

5. In the parallelogram $ABCD$ the circle drawn through A, B, and C cuts AD in A' and DC in C', the lines AD and BC being produced if necessary. Prove $A'D : A'C' :: A'C : A'B$.

6. Two regular polygons of the same number of sides are similar.

7. (*a*) The radius of a circle is 13 inches. Find the side of a square whose area equals that of the circle.

(*b*) In the same circle find the radius of the concentric circle which divides the area of the first circle into two equal parts.

JUNE, 1904

1. What text-book have you used, and how long have you studied this subject?

2. The angle between two secants intersecting without the circumference, the angle between a tangent and a secant, and the angle between two tangents, are each measured by one-half the difference of the intercepted arcs.

3. The straight lines joining the middle points of the adjacent sides of any quadrilateral form a parallelogram.

4. If two straight lines are cut by a series of parallels, the corresponding segments of the two lines are proportional.

5. In a triangle having its base equal to 15 inches, and an area of 60 square inches, a line is drawn parallel to the base through a point 6 inches from the opposite vertex. Find the area of the smaller triangle thus formed.

6. The circumference of a circle is the limit which the perimeters of regular inscribed and circumscribed polygons approach when the number of their sides is doubled indefinitely; and the area of the circle is the limit of the areas of these polygons.

7. When is a line said to be the locus of all points which satisfy a given condition? Show that the locus of a point which moves so that the difference of the squares of its distances from two fixed points remains constant is a line perpendicular to the line joining the fixed points.

8. If the diameter of a given circle is 3 feet, find the diameter of a circle one-fourth as large.

SEPTEMBER, 1904

1. What text-book have you used, and how long have you studied this subject?

2. How is the angle between two intersecting chords of a circle measured? Prove your answer.

3. The trapezoid of which the non-parallel sides are equal is the only trapezoid which can be inscribed in a circle.

4. If through a fixed point within a circle two chords are drawn, the product of the two segments of one is equal to the product of the two segments of the other.

5. If the radius of a circle is 3 feet, what is the area of a sector whose angle is 25°?

6. To construct a square which shall have a given ratio to a given square.

7. When is a circle said to be the locus of a point which satisfies a given condition? Show that the locus of a point which moves so that the sum of the squares of its distances from two fixed points is constant is a circle whose centre is the middle point of the two fixed points.

8. If a line d is added to the radius r of a given circle, how much are the circumference and the area increased?

JUNE, 1905

1. What text-book have you used, and how long have you studied this subject?

2. State and prove the theorem concerning the area of a trapezoid.

3. Given any triangle ABC. Let the bisectors of the interior angle at B and of the exterior angle at C meet at D. Prove angle $BAC =$ twice angle BDC.

4. Two triangles are similar when their homologous sides are proportional.

5. Show that the area of the equilateral triangle inscribed in a circle of radius R equals $\frac{3}{4}R^2\sqrt{3}$.

6. Construct a square equivalent to the sum of any number of given squares. Prove your construction.

7. When is a line said to be the locus of all points satisfying a given condition?

Given AC and BD, two fixed perpendicular diameters of a circle, and P a variable point on the semicircumference BCD. Draw PA intersecting BD at E. Show that the locus of the intersection of BP (produced if necessary) and a line through E parallel to AC is the straight line passing through C and D.

8. What is the area of a circle inscribed in a square whose area is n square feet?

SEPTEMBER, 1905

1. What text-book have you used, and how long have you studied this subject?

2. If two circumferences intersect, the straight line joining their centres bisects their common chord at right angles.

3. Show that the area of an equilateral triangle whose side is a equals $\frac{1}{4}a^2\sqrt{3}$.

4. Rectangles having equal bases are to each other as their altitudes when the latter are commensurable or incommensurable.

5. A, B, C and P are any four points on a circumference. Show that the middle points of PA, PB and PC lie upon a circumference touching the first circumference at P.

6. If from a point without a circle a tangent and a secant are drawn, the tangent is a mean proportional between the whole secant and its external segment.

7. When is a circle said to be the locus of all points satisfying a given condition? Show that a circle is the locus of all points whose distances from two given points are in a given ratio?

8. If a rectangle inscribed in a circle of radius R has one side double the other, show that its area equals $\frac{8}{5}R^2$.

JUNE, 1906

1. What text-book have you used, and how long have you studied this subject?

2. State and prove the theorem concerning the measurement of an angle inscribed in a circle.

3. Two parallel chords of a circle intercept between them equal arcs.

4. If three or more straight lines drawn through a common point intercept two parallels, the corresponding segments of the parallels are proportional.

5. To construct a circumference which shall pass through two given points and be tangent to a given line.

6. In two regular polygons of the same number of sides, two corresponding sides have the same ratio as the radii of the inscribed or circumscribed circles.

7. When is a straight line said to be the locus of all points satisfying a given condition?

Given two intersecting perpendicular lines AB and CD. Construct a number of points, the sum of whose perpendicular distances from AB and CD equals 2 inches. Show that the locus of all such points is the perimeter of a square whose diagonals lie along AB and CD.

8. Calculate the area of the inscribed circle of an equilateral triangle whose side is 3 inches.

SEPTEMBER, 1906

1. What text-book have you used, and how long have you studied this subject?

2. The three perpendicular bisectors of the sides of a triangle meet in a common point. State a construction depending upon this theorem.

3. The sum of two opposite sides of a quadrilateral circumscribed about a circle equals the sum of the other two sides.

4. In an obtuse-angled triangle, the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides increased by twice the product of one of these sides by the projection of the other side upon it.

5. The sides of a right triangle are 10 inches and 5 inches. The hypotenuse of a similar triangle is 10 inches. What is the area of the second triangle?

6. The areas of two triangles which have an angle of one equal to an angle of the other are to each other as the products of the sides including the equal angles.

7. When is a circle said to be the locus of a point satisfying a given condition?

The sides of a right triangle are given in position and the length of the hypotenuse is also given. Find the locus of the middle point of the hypotenuse.

8. If a square and a circle have equal areas, what is the ratio of the side of the square to the radius of the circle?

JUNE, 1907

1. Construct a mean proportion between two given lines; a third proportional to two given lines. Prove your constructions.

2. In any triangle, the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides, minus twice the product of one of these sides and the projection of the other side upon it.

3. The areas of two similar triangles are to each other as the squares of any two homologous sides.

4. The area of a regular polygon is equal to half the product of its apothem and perimeter. State the corresponding theorem on the area of a circle.

5. The difference of the squares of two sides of any triangle is equal to the difference of the squares of the projections of these sides on the third side.

6. When is a circle said to be the locus of all points satisfying a given condition? One side of a triangle is fixed in length and position and the opposite angle is given. The other two sides being variable, find the locus of the movable vertex.

7. The area of a right triangle is 25 square inches and one leg is 10 inches. What is the hypotenuse of a similar right triangle whose area is 50 square inches?

8. A square is inscribed in a given circle. A circle is inscribed in the square. Compare the areas of the two circles.

SEPTEMBER, 1907

1. If two sides of a triangle are unequal, the opposite angles are unequal, and the greater angle is opposite the greater side. If two angles are unequal, the opposite sides are also unequal, and the greater side is opposite the greater angle.

2. In a right triangle a perpendicular is drawn from the vertex of the right angle to the hypotenuse. State and prove three theorems true of the figure thus formed.

3. The bisector of an angle (interior or exterior) of a triangle divides the opposite side into segments which are proportional to the other two sides.

4. Show how to construct a square (*a*) equivalent to a given parallelogram; (*b*) equivalent to a given polygon. Prove your constructions.

5. The perpendiculars from any two vertices of a triangle on the opposite sides are inversely proportional to those sides.

6. When is a straight line said to be the locus of all points satisfying a given condition? Construct a number of points the ratio of whose perpendicular distances from two fixed perpendicular lines is 2. Show that the locus of these points is a straight line.

7. Find the area and circumference of the circumscribed circle of an equilateral triangle whose side is 3 inches.

8. The hypotenuse of an isosceles right triangle is 10 inches. What is the length of the side of the square whose area is double that of the triangle?

JUNE, 1908

1. If two triangles have two sides of one equal respectively to two sides of the other, but the included angle of the first greater than the included angle of the second,

then the third side of the first is greater than the third side of the second. State and prove also the converse theorem.

2. If a chord is drawn through a fixed point within a circle, the product of its segments is the same in whatever direction it is drawn.

3. To construct a square equivalent to the sum of three or more given squares.

4. Two regular polygons of the same number of sides are similar.

5. Opposite angles of a quadrilateral inscribed in a circle are supplementary.

6. When is a straight line said to be the locus of all points satisfying a given condition? Straight lines are drawn through a given point to meet a given straight line which does not pass through the given point. If each of these straight lines is divided in a constant given ratio $m: n$, show that the locus of the point of division is a straight line.

7. The area of an equilateral triangle is 16 square inches. What is the area of its inscribed circle?

8. A square is given. A semicircle is drawn upon one of its sides as a diameter. Within this semicircle a square is constructed with one side resting upon the diameter, the opposite vertices lying in the semicircumference. Find the ratio of the areas of the two squares.

SEPTEMBER, 1908

1. If three or more parallel lines intercept equal parts on one transversal, they intercept equal parts on every transversal.

2. Two triangles are similar when their homologous sides are proportional.

3. To construct a rectangle equivalent to a given square and having the difference of its base and altitude equal to a given line.

4. The perimeters of two regular polygons of the same number of sides are in the same ratio as the radii of the inscribed or circumscribed circles.

5. To construct a circle of a given radius tangent to two given circles.

6. When is a circle said to be the locus of all points satisfying a given condition? Find the locus of the centres of all circles of a given radius which are tangent to a given circle.

7. The altitude of a rectangle is double its base. Its area equals that of an equilateral triangle whose circumscribed circle has a radius of 4 inches. What is the area of the circle described upon a diagonal of the rectangle as a diameter?

8. The sides of a given right triangle are respectively 6 inches and 8 inches. Find the hypotenuse of a similar right triangle if its area is 3 times as great.

JULY, 1909

1. If two triangles have their three sides respectively equal, the triangles are equal.

2. What are similar triangles? In two similar triangles, corresponding altitudes have the same ratio as any two homologous sides.

3. To construct a rectangle equivalent to a given square and having the sum of its base and altitude equal to a given line.

4. The area of a regular polygon is equal to one-half the product of its apothem and perimeter.

5. The base and altitude of an isosceles triangle are equal. A square is constructed whose base rests upon the base of the triangle, and whose upper side terminates in the sides of the triangle. Compare the areas of the square and triangle.

6. Construct a circle of given radius to touch each of two given straight lines.

7. A square is inscribed in a given circle. Within this square is inscribed a second square whose vertices are the middle points of the sides of the first square. Within the second square is inscribed a circle. Compare the circumferences of the two circles; also their areas.

8. The diameter of a circle is 2 inches. Find the area of the circumscribed isosceles triangle whose altitude is 3 inches.

SEPTEMBER, 1909

1. The sum of all the interior angles of any polygon is equal to twice as many right angles as the figure has sides, less four right angles.

2. The angle between two chords which intersect within a circle is measured by one-half the sum of the intercepted arcs.

3. The areas of two similar polygons are in the same ratio as the squares of any two homologous sides.

4. To inscribe in a given circle a square; a regular hexagon.

5. Find the area of an isosceles triangle of altitude 3 inches, which is inscribed in a circle whose diameter is 4 inches.

6. Find the locus of the middle points of all chords of the same length in a fixed circle.

7. The sides of a rectangle are, respectively, 3 inches and 4 inches. Find the diagonal of the similar rectangle whose area is twice as great.

8. Given an equilateral triangle. Draw the inscribed circle C. Draw the inscribed equilateral triangle in C. Draw the inscribed circle of this last triangle. Compare the circumferences of the two circles; also their areas.

JUNE, 1910

1. If two triangles have two sides of one equal to two sides of the other, but the third side of the first greater than the third side of the second, then the angle opposite the third side of the first is greater than the angle opposite the third side of the second.

2. If two straight lines are cut by a series of parallels the corresponding segments of the two lines are proportional.

3. The areas of two similar triangles are to each other as the squares of any two homologous sides.

4. To construct a square equivalent to a given parallelogram.

5. A side of an isosceles right triangle is 4 inches. Find the length of the perpendicular drawn from the vertex of the right angle to the hypotenuse. Construct a square whose area equals the area of the triangle.

6. A straight line is drawn through the point of contact of two tangent circles, forming chords. Radii are drawn to the other extremities of these chords. Show that these radii are parallel.

7. One angle of a right triangle is 60°. Compare the areas of the inscribed and circumscribed circles of the triangle.

8. The areas of a square and an equilateral triangle are equal. Find the ratio of their sides.

SEPTEMBER, 1910

1. The three bisectors of the angles of a triangle meet in a common point.

2. Upon a given straight line to construct a segment which shall contain a given angle.

3. If from a point without a circle a tangent and a secant be drawn, the tangent is a mean proportional between the whole secant and its external segment.

4. The areas of two triangles which have an angle of one equal to an angle of the other are to each other as the products of the sides including those angles.

5. Two triangles ABC and ABC^1 have a common base AB and the vertices C and C^1 lie in a line parallel to AB. A second line parallel to AB cuts AC and BC in M and N, and AC^1 and BC^1 in M^1 and N^1, respectively. Show that $MN = M^1N^1$.

6. A point P is at a distance of 3 inches from a line AB. To construct a circle with a radius of 2 inches passing through P and tangent to AB. How many solutions are there?

7. Given a square. Draw the circumscribing circle. Inscribe in this circle an equilateral triangle. What is the ratio of the areas of the square and the triangle?

8. The sides of a right triangle are respectively 2 and 4 inches. Find the length of the hypotenuse of the similar triangle whose area is half as great.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE, 1886

1. If two angles of a triangle are unequal, the sides opposite are unequal, the greater side lies opposite the greater angle.

2. If CD is the perpendicular from the vertex of the right angle to the hypotenuse of the right triangle ABC, and CE is the bisector of the angle C, prove that the angle DCE is equal to one-half the difference of the angles A and B.

3. The angle between two tangents to a circle is measured by one-half the difference of the intercepted arcs.

4. The diagonals AC and BD of an inscribed quadrilateral intersect at E. If the sides AB and BC subtend arcs of 35° and 103° respectively, and the angle AED between the diagonals is 119°, how many degrees are there in each angle of the quadrilateral?

5. A straight line which divides two sides of a triangle proportionally is parallel to the third side.

6. Two triangles having an angle of one equal to an angle of the other are to each other as the products of the sides including the equal angles.

7. The sum of the perpendiculars from any point within an equilateral triangle to the three sides is equal to the altitude of the triangle.

8. Two regular polygons of the same number of sides are similar.

GEOMETRY (Complete and Preliminary), MAY, 1889

1. The diagonals of a rhombus bisect each other.

2. In the same circle, or in equal circles, the less of two chords is at the greater distance from the centre.

3. A circle is inscribed in a triangle *ABC*. If *D*, *E*, and *F* are the points of contact of the sides *BC*, *CA*, and *AB*, respectively, prove that the angle *EDF* is one-half the supplement of the angle *EAF*.

4. Define similar polygons. Prove that two triangles are similar when they have an angle of the one equal to an angle of the other, and the sides including these angles proportional.

5. Two circles whose diameters are 8 and 2 inches, respectively, touch each other externally. What is the length of their common tangent?

6. Two similar polygons are to each other as the squares of their homologous sides.

7. If *E* and *F* are the middle points of the sides *BC* and *CD* of the parallelogram *ABCD*, prove that the area of the triangle *AEF* is three-eighths the area of the parallelogram.

8. Prove by the Method of Limits that the area of a circle is equal to one-half the product of its circumference and radius.

GEOMETRY (Complete), SEPTEMBER, 1889

1. The perpendicular from the vertices of a triangle to the opposite sides meet in a common point.

2. If *AE* is the bisector of the angle *A* of the triangle *ABC*, and *AF* is drawn perpendicular to *AE* at *A*, prove that *AF* bisects the exterior angle at *A*.

3. A straight line perpendicular to a radius at its extremity is tangent to the circumference.

4. A circle is described on the line AB as a diameter, and AC is drawn equal and perpendicular to AB. Prove that the line BC is bisected by the circumference.

5. The perimeters of two similar polygons are in the same ratio as any two homologous sides.

6. The area of a parallelogram is equal to the product of its base and altitude.

7. The perimeter of a rhombus is 52, and its least diagonal is 10. What is its area?

8. Define a regular polygon. Prove that two regular polygons of the same number of sides are similar.

JUNE, 1890

1. Two angles whose sides are perpendicular each to each, are either equal or supplementary.

2. If two triangles have two sides of one equal respectively to two sides of the other, but the third side of the first greater than the third side of the second, the included angle of the first is greater than the included angle of the second.

3. If the two diagonals of a quadrilateral are equal, and bisect each other at right angles, the figure is a square.

4. In equal circles, or in the same circle, the greater of two chords subtends the greater arc.

5. The length of the straight line joining the middle points of the non-parallel sides of a circumscribed trapezoid is equal to one-fourth the perimeter of the trapezoid.

6. Two triangles are similar when they are mutually equiangular.

7. The areas of two similar triangles are to each other as the squares of their homologous sides.

8. If the radius of a circle is 6, what is the area of a segment whose arc is 60°?

(Take $\pi=3.1416$.)

SEPTEMBER, 1890

1. Two right triangles are equal when the hypotenuse and a side of one are equal respectively to the hypotenuse and a side of the other.

2. If two angles of a triangle are unequal, the sides opposite are unequal, and the greater side lies opposite the greater angle.

3. If *E* is the middle point of the side *BC* of the parallelogram *ABCD*, prove that *AE* and *BD* trisect each other.

4. In the same circle, or in equal circles, the less of two chords is at the greater distance from the centre.

5. If a series of parallels, cutting two straight lines, intercept equal distances on one of the lines, they also intercept equal distances on the other line.

6. If a polygon is circumscribed about a circle, the area of the polygon is equal to one-half the product of its perimeter by the radius of the circle.

7. To construct a square equivalent to the difference of two given squares.

8. Regular hexagons are inscribed within, and circumscribed about, a given circle. If the apothem of the circumscribed hexagon is 12, what is the apothem of the inscribed hexagon?

SEPTEMBER, 1891

1. The perpendicular is the shortest line which can be drawn from a point to a straight line.

2. The straight line joining the middle joints of two sides of a triangle is parallel to the third side and equal to one-half of it.

3. Two parallel tangents intercept equal arcs on the circumference of a circle.

4. If *AD* and *BE* are the perpendiculars from the vertices A and B, respectively, of the triangle ABC, to the opposite sides, and intersect at O, prove that $OA : OE :: OB : OD$.

5. Two triangles are similar when the sides of one are perpendicular, respectively, to the sides of the other.

6. In an obtuse-angled triangle, the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides, increased by twice the product of one of these sides and the projection of the other side upon it.

7. The area of a trapezoid is equal to one-half the sum of its parallel sides multiplied by its altitude.

8. If the diagonals AC and BG of the regular octagon $ABCDEFGH$ intersect at O, how many degrees are there in the angle AOB?

JUNE, 1892

1. Two triangles are equal when the three sides of one are equal respectively to the three sides of the other.

2. In the same circle, or in equal circles, equal chords are equally distant from the centre.

3. If AC and BC are tangents to a circle whose centre is O, from a point C without the circle, prove that the centre of the circle which passes through O, A, and B, bisects OC.

4. Two triangles are similar when their homologous sides are proportional.

5. Two triangles having an angle of one equal to an angle of the other are to each other as the products of the sides including the equal angles.

6. Define a segment of a circle; equivalent triangles. When are two polygons called mutually equilateral?

7. AD and BC are the parallel sides of the trapezoid $ABCD$, whose diagonals intersect at O. Prove

$$\text{area } AOD : \text{area } BOC = \overline{AO}^2 : \overline{OC}^2.$$

8. If the circumference of a circle is divided into any number of equal arcs, their chords form a regular polygon inscribed in the circle.

SEPTEMBER, 1892

1. If two straight lines are drawn from a point within a triangle to the extremities of any side, the angle included by them is greater than the angle included by the other two sides.

2. The perpendiculars from the vertices of a triangle to the opposite sides meet in a common point.

3. In the same circle, or in equal circles, the less of two chords is at the greater distance from the centre.

4. An angle formed by a tangent and a chord is measured by one-half its intercepted arc.

5. The diagonals AC and BD of the quadrilateral $ABDC$ intersect at E. If $AE:EC=BE:ED$, prove that the triangles AEB and DEC are similar.

6. If through a fixed point without a circle a secant and a tangent are drawn, the product of the whole secant and its external segment is equal to the square of the tangent.

7. In an obtuse-angled triangle, the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides increased by twice the product of one of these sides and the projection of the other side upon it.

8. A regular hexagon $ABCDEF$ is inscribed in a circle whose radius is 2; find the length of the diagonal AC.

Define an inscribed angle; a diagonal of a polygon; similar triangles.

JUNE, 1893

1. If oblique lines are drawn from a point to a straight line, two oblique lines cutting off equal distances from the foot of the perpendicular are equal; and of two oblique lines cutting off unequal distances from the foot of the perpendicular, the more remote is the greater.

2. The straight line joining the middle points of the non-parallel sides of a trapezoid is parallel to the bases and equal to half their sum.

3. In equal circles, or in the same circle, the greater of two chords subtends the greater arc.

4. State and prove the converse of the preceding.

If the diameter of a circle is 3 inches, what is the length of an arc of 80°?

5. If AB and CD are equal chords of a circle and intersect at E, prove that $AE=ED$ and $BE=EC$.

6. If any two chords are drawn through a fixed point in a circle, the product of the segments of one is equal to the product of the segments of the other.

7. AD and BC are the parallel sides of a trapezoid $ABCD$, whose diagonals intersect at E. If F is the middle point of BC, prove that EF produced bisects AD.

8. Two similar triangles are to each other as the squares of their homologous sides.

SEPTEMBER, 1893

1. If two sides of a triangle are unequal, the angles opposite are unequal, and the greater angle lies opposite the greater side.

2. State and prove the converse of the preceding.

Define a convex polygon; a segment of a circle; equivalent polygons; similar sectors.

3. An inscribed angle is measured by one-half of its intercepted arc.

4. If E and F are the middle points of the non-parallel sides AB and CD, respectively, of the trapezoid $ABCD$, and EF cuts AC at G and BD at H, prove $EG=FH$.

5. A straight line which divides two sides of a triangle proportionally is parallel to the third side.

6. AB and AC are tangents at the points B and C to the circumference whose centre is O; prove that the angle OBC is one-half the angle A.

7. The area of a parallelogram is equal to the product of its base and altitude.

8. The perimeters of two regular polygons of the same number of sides are to each other as the radii of their circumscribed or of their inscribed circles; and their areas are to each other as the squares of these radii.

JUNE, 1894

(Omit ONE of the first seven questions. Be sure to state all reasons.)

1. Prove that if two straight lines are cut by a secant line making the alternate interior angles equal, the two lines are parallel.

2. Prove that two parallelograms are equal when two adjacent sides and the included angle of one are equal, respectively, to adjacent sides and the included angle of the other.

3. Prove that two triangles having an angle of the one equal to an angle of the other, are to each other as the products of the sides including the equal angles.

4. Prove that two rectangles having equal altitudes are to each other as their bases, even when their bases are incommensurable.

5. Prove that a line perpendicular to a tangent, at its point of contact, passes through the centre of the circle.

6. Show that two isosceles triangles will be similar if their vertical angles are equal; and that if their vertical angles are unequal, they cannot be similar.

7. What is the locus of a point which moves so that the tangent drawn from it to a fixed circle is of constant length? Prove.

8. The perimeter of a trapezoid is 56 inches. If each of the non-parallel sides is 13 inches long, and the area is 180 square inches, what are the respective lengths of the parallel sides?

SEPTEMBER, 1894

(*Be sure to state all reasons. Omit ONE of the first seven questions.*)

1. A straight line perpendicular to one of two parallels is perpendicular to the other.

2. Any point in the bisector of an angle is equally distant from the sides of the angle.

3. If the diagonals of a parallelogram are equal, the figure is a rectangle.

4. A parallel to one side of a triangle divides the other two sides proportionally, even when the segments of each side are incommensurable.

5. Two polygons are similar when they are composed of the same number of triangles, similar each to each, and similarly placed.

6. Two sides and the included angle of a quadrilateral are fixed. If the angle opposite the included angle is its supplement, show that the locus of the fourth vertex of the quadrilateral is the circumference of a circle passing through the three given vertices.

7. AB is the common chord of two equal circles. Show that two tangents drawn from C (a point on AB extended) one to each of the two circles, will be equal.

8. The altitude of an isosceles triangle is 6. Its perimeter is 36. What is its area?

JUNE, 1895

(Every reason must be stated in full.)

1. If two triangles have two sides of one equal, respectively, to two sides of the other, but the third side of the first greater than the third side of the second, the included angle of the first is greater than the included angle of the second.

2. In any triangle the bisector of an angle divides the opposite side into segments proportional to the adjacent sides.

3. The perpendicular erected at the middle point of a chord passes through the centre of the circle, and bisects the arc subtended by the chord.

4. Two sides of a triangle are, respectively, 10 and 15. The area is 45. Find the length of the third side.

5. Prove the formula for the area of a regular polygon.

6. The circumferences of two circles are to each other as their radii.

7. AB is the hypotenuse of a right triangle, and D and E are the middle points of BC and AC, respectively. Prove $(2\overline{AD})^2 + (2\overline{BE})^2 = 5\overline{AB}^2$.

8. AB is a given straight line. At each point of AB a perpendicular is erected of the same length as its distance from A. What is the locus of the upper extremities of these perpendiculars? Prove your statement.

JUNE, 1896

(*Every reason must be stated in full.*)

1. If straight lines are drawn to the extremities of a straight line from any point in the perpendicular erected at its middle point, they make equal angles with the line and with the perpendicular.

2. Two right triangles are equal when the hypotenuse and a side of one are equal, respectively, to the hypotenuse and a side of the other.

3. Prove the formula for the sum of the angles of any polygon. Define a regular polygon. How many degrees in each angle of a regular heptagon?

4. In the same circle or in equal circles chords equally distant from the centre are equal.

5. Two triangles are similar when their homologous sides are proportional.

6. A hexagon is formed by joining in succession the middle points of the sides of a given regular hexagon. Find the ratio of the areas of these two hexagons.

7. If AB and A_1B_1 are any two chords of the outer of two concentric circles, which intersect the circumference of the inner circle at P, Q, and P_1, Q_1, respectively, prove $AP \cdot PB = A_1P_1 \cdot P_1B_1$.

JULY, 1897

(*All reasons must be stated in full.*)

1. Two and only two equal oblique lines can be drawn from a given point to a given straight line.

2. The perpendiculars from the vertices of a triangle to the opposite sides meet at a common point.

3. A line perpendicular to a tangent at its point of contact passes through the centre of the circle.

4. Define equal figures; equivalent figures; similar figures. Any two polygons are similar if they are composed of the same number of triangles, similar each to each and similarly placed.

5. State and prove the converse of the proposition in the previous question.

6. If the points midway between the vertices and the centre of a parallelogram be joined, prove that the quadrilateral formed will be a parallelogram similar to the given parallelogram, and equivalent to one-fourth of it.

7. Find the length of the side of an equilateral triangle which shall be equivalent to a regular hexagon which is circumscribed about a circle of radius equal to 6 inches.

8. BB' is the diameter of a circle of which the centre is C. On $B'B$ produced through B, A is so taken that $AB = BC$. From A a line is drawn to P, any point of the circle. If D bisects the line AP, prove that the locus of D is a circle with its centre at B.

SEPTEMBER, 1897

(All reasons must be stated in full.)

1. Two triangles are equal when the three sides of one are respectively equal to the three sides of the other.

2. The medians of a triangle meet at a common point.

3. A line which divides two sides of a triangle proportionally is parallel to the third side.

4. Prove that the angle between two secants intersecting without the circumference is measured by one-half the difference of the intercepted arcs.

What is the *measure* of a geometrical quantity, and does this definition agree with your proof?

5. The base BC of a triangle ABC is a given fixed line, and the vertex A moves so that the area of ABC shall

always be equal to *K*. Draw the locus of *A*, and prove your construction to be correct.

6. From *A* tangents are drawn to a circle with centre at *C*, touching it at *B* and *D*. If *BD* intersects *AC* at *E*, prove that the radius of the circle is a mean proportional between *CA* and *CE*.

7. If any two chords are drawn through a fixed point within a circle, the product of the segments of one chord is equal to the product of the segments of the other.

A is a point within a circle of radius 7 inches. One of the chords through *A* is divided at *A* into segments 3 and 8 inches long respectively. How far is *A* from the centre of the circle?

JULY, 1898

(*Every reason must be stated in full.*)

1. If two parallels are cut by a third straight line, the alternate interior angles are equal.

2. Two triangles are equal when a side and two adjacent angles of one are equal respectively to a side and two adjacent angles of the other.

3. If an exterior angle is formed at the vertex of an isosceles triangle, its bisector is parallel to the base.

4. In equal circles, or in the same circle, equal chords subtend equal arcs.

5. An angle formed by a tangent to a circle and a chord is measured by half its intercepted arc.

6. In any triangle the bisector of an angle divides the opposite side into segments proportional to the adjacent sides.

7. The circumferences of two circles are to each other as their radii, and their areas are to each other as the squares of their radii.

8. If the diagonals AC and BG of the regular octagon $ABCDEFGH$ intersect at O, how many degrees are there in the angle AOB?

SEPTEMBER, 1898

(Every reason must be stated in full.)

1. If two straight lines are cut by a third so as to make the alternate interior angles equal, the two lines are parallel.

2. If two triangles have two sides of one equal respectively to two sides of the other, but the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second.

3. The diagonals of a rhombus bisect each other at right angles.

4. In the same circle, or in equal circles, equal chords are equally distant from the centre.

5. The angle between a secant and a tangent is measured by one-half the difference of the intercepted arcs.

6. Any two rectangles are to each other as the products of their bases by their altitudes.

7. The area of a circle is equal to one-half the product of its circumference and radius.

8. A regular hexagon $ABCDEF$ is inscribed in a circle whose radius is 4. Find the length of the diagonal AC.

JUNE, 1899

(The reason for each step must be stated in full.)

1. Two parallel lines are everywhere equally distant.

2. If two angles of a triangle are unequal, the sides opposite are unequal, and the greater side lies opposite the greater angle.

3. The sum of the lines drawn from any point within a triangle to the vertices is greater than the half sum of the three sides.

4. An angle inscribed in a circle is measured by one-half its intercepted arc.

5. If any two chords are drawn through a fixed point in a circle, the rectangle of the segments of one is equal to the rectangle of the segments of the other.

6. Two triangles having an angle of one equal to an angle of the other, are to each other as the rectangles of the sides including the equal angles.

7. Two circles which touch each other externally at C have a common tangent whose points of contact are A and B. Prove that ACB is a right angle.

8. The lines joining the middle points of opposite sides of a quadrilateral bisect each other.

SEPTEMBER, 1899

(*The reason for each step must be stated in full.*)

1. Two angles whose sides are perpendicular each to each are either equal or supplementary.

2. If two straight lines are drawn from a point within a triangle to the extremities of any side, the angle included by them is greater than the angle included by the other two sides.

3. The lines joining the middle points of the adjacent sides of a quadrilateral form a parallelogram whose perimeter is equal to the sum of the diagonals of the quadrilateral.

4. The angle between two secants of a circle intersecting without the circumference is measured by one-half the difference of the intercepted arcs.

5. Two polygons are similar when they are composed of the same number of triangles, similar each to each and similarly placed.

6. Prove that the side of an equilateral triangle inscribed in a circle is equal to the radius of the circle multiplied by $\sqrt{3}$.

7. If the bisectors of the interior angle at C and the exterior angle at B of the triangle ABC meet at D, prove that the angle BDC is equal to $\frac{1}{2}A$.

8. If the opposite angles of a quadrilateral are supplementary, the quadrilateral can be inscribed in a circle.

JUNE, 1900

1. From a given point without a straight line, only one perpendicular can be drawn to the line.

2. If two triangles have two sides of one equal respectively to two sides of the other, but the third side of the first greater than the third side of the second, the included angle of the first is greater than the included angle of the second.

3. The medians of a triangle meet in a point.

4. A straight line can intersect a circumference in not more than two points.

5. In the same circle, or in equal circles, two central angles are in the same ratio as their intercepted arcs.

6. Two triangles are similar when their sides are parallel each to each, or perpendicular each to each.

7. If the triangle ABC has the side AB fixed, and the side AC twice the side BC, the vertex C lies on a circle which divides the line AB externally and internally in the same ratio.

8. Compute the difference in area between a circle of radius 3 and an inscribed regular hexagon.

SEPTEMBER, 1900

1. Two angles whose sides are perpendicular, each to each, are either equal or supplementary.

2. The line joining the middle points of the non-parallel sides of a trapezoid is parallel to the bases, and equal to half their sum.

3. A tangent to a circle is perpendicular to the radius drawn to the point of contact.

4. If any two chords be drawn through a fixed point within a circle the product of the segments of one chord is equal to the product of the segments of the other.

5. Two rectangles having equal altitudes are to each other as their bases.

6. Two similar polygons are to each other as the squares of their homologous sides.

7. The line joining the middle points of the arcs subtended by the sides AB and AC of an inscribed triangle cuts AB at F, and AC at G. Prove that $AF = AG$.

8. An angle at the centre of a circle is subtended by an arc equal to the radius of the circle. Compute the number of degrees in the angle.

JUNE, 1901

1. The perpendiculars from the vertices of a triangle to the opposite sides meet in a common point.

2. A circle is inscribed in a triangle ABC. If D, E, and F are the points of contact of the sides BC, CA; and AB, respectively, prove that the angle EDF is one-half the supplement of the angle EAF.

3. Two circles whose diameters are 8 and 2 inches, respectively, touch each other externally. What is the length of their common tangent?

4. Define similar polygons. Show that the perimeters of two similar polygons are in the same ratio as any two homologous sides.

5. In equal circles or in the same circle the greater of two chords subtends the greater arc.

6. Show how to construct a triangle equivalent to a given polygon.

7. If two triangles have two sides of the one equal, respectively, to two sides of the other, but the third side of the first greater than the third side of the second, the included angle of the first is greater than the included angle of the second.

8. If the radius of a circle is 6, what is the area of a segment whose arc is 60°?

SEPTEMBER, 1901

1. Two right triangles are equal when the hypotenuse and a side of one are equal respectively to the hypotenuse and a side of the other.

2. If two angles of a triangle are unequal, the sides opposite are unequal, and the greater side lies opposite the greater angle.

3. If E is the middle point of the side BC of the parallelogram $ABCD$, prove that AE and BD trisect each other.

4. The angle between two tangents to a circle is measured by one-half the difference of the intercepted arcs.

5. If a series of parallels, cutting two straight lines, intercept equal distances on one of those lines, they also intercept equal distances on the other line.

6. Show how to construct a square equivalent to the difference of two given squares.

7. In any triangle the product of any two sides is equal to the diameter of the circumscribed circle multiplied by the perpendicular drawn to the third side from the vertex of the opposite angle.

8. Regular hexagons are inscribed within, and circumscribed about a given circle. If the apothem of the circumscribed hexagon is 10, what is the apothem of the inscribed hexagon?

JUNE, 1902

1. From a point without a straight line, one and only one perpendicular can be drawn to the line.

2. The bisector of an angle of a triangle divides the opposite side into segments which are proportional to the adjacent sides.

3. If from any point without a circle tangents are drawn, the angle contained by the tangents is double the angle contained by the line joining the points of contact and the diameter drawn through one of them.

4. Show how to construct a polygon similar to two given polygons and equivalent to their sum.

5. A line perpendicular to a radius at its extremity is tangent to the circle. Prove also the converse of this theorem.

6. The perimeter of an isosceles triangle whose base is its shortest side is 100 decimeters; the difference between the base and an adjacent side is 23 decimeters. What is the altitude of the triangle, and what is its area? Determine also the area in square inches.

7. A circle is equivalent to one-half the rectangle formed by its circumference and radius.

SEPTEMBER, 1902

1. If two parallel straight lines are cut by a third straight line, the alternate interior angles are equal.

2. Similar polygons are to each other as the squares upon their homologous sides.

3. If two circles are tangent externally, their common exterior tangent is a mean proportional between the diameters of the circles.

4. Show how to describe upon a given straight line a segment of a circle which shall contain a given angle.

5. Two triangles are similar if the sides of one are proportional to the sides of the other, each to each.

6. Three equal circles of a radius of 12 feet are drawn tangent to each other. What is the area between them? Express area in square feet and in square meters.

7. If a regular polygon is circumscribed about a circle, and a similar polygon is inscribed in the circle, and if the number of their sides is indefinitely increased, show that their perimeters approach the circumference of the circle as a common limit.

JUNE, 1903

1. If two sides of a triangle are unequal, the angles opposite are unequal, and the greater angle lies opposite the greater side; prove also the converse.

2. Define a quadrilateral, also a trapezium. Show that if the non-parallel sides of a trapezoid are equal, the angles which they make with the bases are equal.

3. In the same circle or in equal circles the less of two chords is at the greater distance from the centre.

4. A line parallel to one side of a triangle divides the other two sides proportionally.

5. The area of a rhombus is 96, and its side is 10; find the lengths of its diagonals.

6. If a medial line is drawn from the vertex to the base of a triangle, the sum of the squares of the other two sides is equal to twice the square of half the base increased by twice the square of the medial line.

7. In a square whose side is 2, inscribe a regular octagon having two of its vertices in each side of the square, and find the length of a side of the octagon.

8. The perimeters of two regular polygons of the same number of sides are to each other as the radii of their circumscribed or of their inscribed circles; and their areas are to each other as the squares of these radii.

SEPTEMBER, 1903

1. The straight line joining the middle points of the non-parallel sides of a trapezoid is parallel to the bases and equal to one-half their sum.

2. If one acute angle of a right triangle is double the other, the hypotenuse is double the shortest side.

3. An inscribed angle is measured by one-half its intercepted arc.

4. Two triangles are similar when the sides of one are perpendicular respectively to the sides of the other.

5. C and D are the middle points of a chord AB and its subtended arc. If $AD=9$ and $CD=3$, what is the diameter of the circle?

6. In the triangle ABC let C be an acute angle; what is the expression for the square of AB, in terms of the two other sides, and the projection of one side upon the other? Prove your statement.

7. If the radius of a circle is 5, find the area of the sector whose central angle is 50°.

8. If the number of sides of a regular inscribed and of a regular circumscribed polygon is indefinitely increased, the area of each polygon approaches the area of the circle as a limit.

JUNE, 1904

1. (*a*) In an isosceles triangle the angles opposite the equal sides are equal.

(*b*) State and prove the converse proposition.

2. Through any three points not in the same straight line one and only one circumference can be passed.

3. If two triangles have an angle of one equal to an angle of the other, and the including sides proportional, the triangles are similar.

4. The square described on the hypotenuse of a right triangle is equivalent to the sum of the squares on the other two sides.

5. Take ABC any triangle in which the side AB is not equal to AC. Lay off on AB a distance $AC' = AC$ and on AC a distance $AB' = AB$ and draw $B'C'$. If $B'C'$ intersect BC in D, prove that AD bisects the angle A.

6. Let A be the point of intersection of two circles. Through A draw any secant intersecting the circles again in M and N. Prove that the tangents to the circles at M and N intersect in a constant angle (i.e., in an angle which is the same for all the secants drawn through A).

7. Compute the areas of the two segments of a circle of radius $2\sqrt{3}$ inches made by a chord which is 3 inches from the centre of the circle.

SEPTEMBER, 1904

1. (*a*) If two triangles have two sides of one equal, respectively, to two sides of the other and the included angles unequal, the triangle with the greater included angle has the greater third side.

(*b*) If two triangles have two sides of one equal, respectively, to two sides of the other and the third sides unequal, the triangle with the greater third side has the greater included angle.

2. An angle inscribed in a circle is measured by half its intercepted arc.

3. The bisector of an angle of a triangle divides the opposite side into segments proportioned to the adjacent sides.

4. The area of a circle is equal to half the product of its circumference and its radius.

5. ABC is a triangle inscribed in a circle with centre O. Take D the middle point of the arc BC and draw OD and AD. Prove that the angle ADO equals half the difference of the angles B and C.

6. If a line is drawn from the vertex A of a triangle ABC to any point D of the opposite side, and any point O on AD is joined to B and C, prove $\frac{\triangle ABC}{\triangle OBC}=\frac{AD}{OD}$.

7. From a point at a distance of 10 inches from the centre of a circle of radius 5 inches, two tangents are drawn. Compute the area bounded by the tangents and their included arc.

JUNE, 1905

1. Two triangles are equal when the three sides of one are equal respectively to the three sides of the other.

2. An angle formed by a tangent to a circle and a chord through the point of tangency is measured by one-half the intercepted arc.

3. Two circles are tangent internally. If chords of the larger circle are drawn from the point of tangency, prove that they are divided proportionally by the smaller circle.

4. If a straight line divides two sides of a triangle proportionally it is parallel to the third side.

5. The areas of two regular polygons of the same number of sides are to each other as the squares of their radii or their apothems.

6. Prove that if one acute angle of a triangle is double another the triangle can be divided into two isosceles triangles by a straight line drawn through the vertex of the third angle.

7. Given a regular hexagon each side of which is 6 inches. With three of the alternate vertices as centres arcs of circles are drawn passing through the centre of the hexagon. Find the area of the three loops thus formed.

SEPTEMBER, 1905

1. Two parallelograms are equal if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

2. If two triangles have their sides respectively proportional they are similar.

3. Through one of the points of intersection of two circles a diameter of each circle is drawn. Show that the line joining the other ends of these diameters goes through the other point of intersection.

4. The area of an inscribed regular hexagon is a mean proportional between the areas of the inscribed and circumscribed equilateral triangles.

5. In the same circle or in equal circles two central angles have the same ratio as their intercepted arcs.

6. Three equal circles are described each tangent to the other two. If the common radius is R, find the area contained between the circles.

JUNE, 1906

(Time allowed, one hour and three-quarters.)

1. Prove: In the same circle or in equal circles equal chords are equally distant from the centre; and of two unequal chords the less is at the greater distance from the centre.

2. Prove: If two polygons are composed of the same number of triangles, similar each to each and similarly placed, the polygons are similar.

3. Prove: The areas of two triangles which have an angle of one equal to an angle of the other are to each other as the products of the sides including the equal angles.

4. Prove that if two altitudes of a triangle are equal the triangle is isosecles.

5. From any point C on the circumference of a circle with its centre at O drop a perpendicular CD to a fixed diameter AB of the circle and lay off on the radius OC a segment OM equal to OD. Show that the locus of M is a circle.

6. How much longer is the circumference of a circle of radius 10 than the perimeter of an inscribed regular octagon? Find the result to one place of decimals.

SEPTEMBER, 1906

(Time allowed, one hour and three-quarters.)

1. Prove: Two triangles are equal when the three sides of one are equal respectively to the three sides of the other.

2. Prove: If two secants intersect without a circle, the whole secants and their external segments are reciprocally proportional.

3. Prove: Regular polygons of the same number of sides are similar.

4. Prove that the bisectors of the angles of a quadrilateral form a second quadrilateral of which the opposite angles are supplementary.

5. Through a point M in the base of a triangle parallels to the other two sides of the triangle are drawn, forming a parallelogram. Find the locus of the centre of this parallelogram as the point M moves along the base of the triangle.

6. Find to one place of decimals the area of a six-pointed star formed by joining the alternate vertices of a regular hexagon inscribed in a circle of radius 3 inches.

JUNE, 1907

(Time, one hour and three-quarters.)

1. Prove: The angle between two secants intersecting without the circumference is measured by one-half the difference of the intercepted arcs. State the corresponding propositions for the angle between a tangent and a secant and the angle between two tangents.

2. Prove: The bisector of an exterior angle of a triangle meets the opposite side produced in a point whose distances from the extremities of that side are proportional to the other two sides.

3. Prove: The areas of two rectangles having equal altitudes are to each other as their bases.

4. A point moves so that the sum of its distances from two given intersecting straight lines is always the same. Prove that its locus is a straight line which forms with the given lines an isosceles triangle.

5. A point moves so that the angle made by the two lines which connect it with two fixed points A and B is always the same. What is its locus?

6. A cow is tied to a stake, which is 15 feet from a straight fence, by a rope of such length that she may graze at any distance not greater than 30 feet from the stake. Over how many square feet may she graze? (Find area to nearest square foot.)

SEPTEMBER, 1907

(Time, one hour and three-quarters.)

1. Prove: The line joining the middle points of the non-parallel sides of a trapezoid is parallel to the bases and equal to half their sum.

2. Prove if in the triangle ABC the line drawn from the vertex C to the middle of the opposite side is equal to half the latter, the area of the triangle is equal to half the product of AC and BC.

3. Prove: An angle between two chords which intersect within a circumference is measured by one-half the sum of its intercepted arc and the arc intercepted by its vertical angle.

Two chords which intersect within a circumference divide the latter into arcs which, taken in order, are proportional to 1, 2, 3, 4. What angles do the chords make with each other?

4. When are two polygons similar? Prove: Two triangles are similar when their homologous sides are proportional.

5. Given a fixed point, D, within a triangle, ABC. Take any point, E, on the perimeter of the triangle, draw DE, and let P be the middle point of DE. Find the locus of P when E traces out the whole perimeter of the triangle. Describe the locus exactly, and prove that your answer is correct.

6. A belt passes around two circular wheels whose radii are 8 and 3 feet, respectively, and whose centres are 13 feet apart. Find to one decimal place the length of the belt when drawn tight, assuming that the angle between the radii drawn from the points of tangency of the belt with either wheel to the centre of the corresponding wheel is 135°.

JUNE, 1908

(Time, one hour and three-quarters.)

1. Prove: If two triangles have the three sides of the one equal, respectively, to the three sides of the other, the triangles are equal.

2. Prove: The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.

3. Prove: The bisector of the vertical angle of a triangle divides the base into segments proportional to the adjacent sides.

4. Given a circle and a straight line. Show how to construct a circle which shall be tangent to the given circle at a given point and also tangent to the straight line. Prove the construction correct.

5. A regular hexagon is circumscribed about, and a square is inscribed in, a circle of radius 1. Find the difference between the perimeter of the circle and half the sum of the perimeters of the two polygons, accurate to thousandths.

6. A parallelogram *ABCD* has the vertex *A* fixed and the directions of the adjacent sides *AB* and *AD* also fixed. Find the locus of the vertex *C* if the sum of the two sides *AB* and *AD* is constant, and prove your answer correct.

SEPTEMBER, 1908

(Time, one hour and three-quarters.)

1. Prove: In the same circle, or in equal circles, the greater of two chords subtends the greater arc, each arc being less than a semicircumference.

2. If perpendiculars are drawn from any point in the base of an isosceles triangle to the sides, prove that the sum of these perpendiculars is equal to the perpendicular drawn from either extremity of the base to the opposite side.

3. If the opposite angles of a quadrilateral are supplementary, prove that it can be inscribed in a circle.

4. Define similar polygons. Prove: The areas of two similar polygons are to each other as the squares of their homologous sides.

5. In a circle of radius 2 inches find correct to two decimal places the area of a segment which subtends an angle of 60° at the centre.

6. If a variable circle is always tangent to a fixed circle and to a fixed straight line, and if the straight line joining the points of tangency is produced to meet the circumference of the fixed circle again, prove that this point of intersection is a fixed point.

JUNE, 1909

(Time, one hour and three-quarters.)

1. If two triangles have two sides of one equal respectively to two sides of the other, but the included angle in the first greater than the included angle in the second, prove that the third side of the first is greater than the third side of the second.

2. If two medians be drawn from two vertices of a triangle and produced their own length beyond the opposite sides, and these extremities be joined to the third vertex, prove that these two lines will be equal in length and lie in the same straight line.

3. Prove that two parallel lines which cross a circle intercept equal arcs on its circumference.

4. The distance between the centres of two circles, whose radii are 25 inches and 17 inches, respectively, is 28 inches. Find the lengths of the segments into which their common chord divides the line joining their centres, and the length of their common chord.

5. Prove that two triangles which have their sides proportional are similar.

6. With the vertices of a regular hexagon whose side is 20 inches in length as centres and a radius of 10 inches, six arcs are described within the hexagon. Assuming $\pi = 3.14$, find the perimeter and the area of the figure bounded by these arcs.

SEPTEMBER, 1909

(Time, one hour and three-quarters.)

1. If two angles of a triangle are unequal, prove that the opposite sides are unequal, and that the greater side is opposite the greater angle.

2. A tangent of constant length is drawn to a given circle. As the point of tangency moves around the circumference of the circle, find the locus of the other end of the tangent, and prove the correctness of your construction.

3. If two chords intersect within a circle, prove that the product of the segments of one chord is equal to the product of the segments of the other.

4. (*a*) Given a triangle whose base is 8 inches and whose altitude is 6 inches, how far from the vertex must a line be drawn parallel to the base so as to divide the triangle into two parts of equal area?

(*b*) Trisect a right angle, and explain your construction.

5. If two triangles have an angle of the one equal to an angle of the other, and the including sides proportional, prove that the two triangles are similar.

6. The angle of a sector of a circle of radius 5 feet is 36°. Assuming $\pi = 3.14$, find the area of a regular hexagon whose perimeter is equal to the length of the arc of the sector.

JUNE, 1910

(Time, one hour and three-quarters).

1. Prove that two triangles are similar, when the sides of one are parallel respectively to the sides of the other.

2. The tangents from a point P to two given circles which do not intersect are equal. Prove that the circle drawn with centre P, and passing through the points of tangency, will cut each of the given circles at right angles.

N. B. Two circles cut at right angles if the tangents at their point of intersection are perpendicular to each other.

3. Prove that if two straight lines are cut by a third, making the alternate interior angles equal, the two lines are parallel.

State, without proof, the converse of the above theorem.

4. If E and F are the middle points of the sides AB and AC of a triangle, and D any point in BC, prove that the area of the quadrilateral $AEDF$ is equal to one-half the area of the triangle ABC.

5. If any number of angles are inscribed in the same segment of a circle, prove that their bisectors meet in a common point.

6. A hexagon is formed by joining in succession the middle points of the sides of a given regular hexagon, the perimeter of which is 6 feet. Find, correct to three decimal places, the difference between the areas of the two hexagons.

GEOMETRY FOR SPECIAL STUDENTS IN ARCHITECTURE

JUNE, 1910

1. (*a*) What is the locus of points equidistant from two fixed points?

(*b*) What is the locus of points at a given distance from a fixed point?

(*c*) Construct a point which is both equidistant from two fixed points A and B, 4 inches apart, and at a distance of 3 inches from a fixed point C.

2. A circle is inscribed in an equilateral triangle whose side is 5 inches. Find the areas of the triangle and of the circle.

3. Find the perimeter of a regular trapezoid whose bases are 20 inches and 10 inches, and whose altitude is 5 inches.

4. How would you find the centre of a sphere that is to be inscribed in a regular tetrahedron $ABCD$?

5. What is the weight of a brick cupola in the form of a hemispherical shell, 6 inches thick and 15 feet in inner diameter, if the weight per cubic foot is 50 pounds?

6. A lateral edge of a regular pyramid measures 12 inches, and each edge of its square base measures 4 inches. Find its total surface.

SEPTEMBER, 1910

(Time, one hour and three-quarters.)

1. If through a fixed point without a circle a secant and a tangent are drawn, prove that the tangent will be a mean proportional between the whole secant and its external segment.

2. If E is the middle point of CD, one of the non-parallel sides of a trapezoid $ABCD$, prove that the area of the triangle ABE is equal to one-half the area of the trapezoid.

3. Prove that two angles whose sides are perpendicular each to each are either equal or supplementary. State the converse theorem. Is the latter true?

4. An equilateral triangle ABC has sides each 4 inches long. The points of bisection of the sides are connected by arcs of circles having as centres A, B, and C, respectively. Find, correct to three decimal places, the area left when the circular sectors are removed from the triangle.

5. Given two sides of a triangle and the angle opposite one of them, show how to construct the triangle.

6. In a right triangle ABC, of which C is the right angle, the sides AC and BC are respectively bisected by two fixed points, P and Q. Find the locus of C for all positions of the triangle.

GEOMETRY FOR SPECIAL STUDENTS IN ARCHITECTURE, SEPTEMBER, 1910

1. Show how to (*a*) inscribe a circle within a triangle: (*b*) circumscribe a circle about a triangle.

2. A hexagon is inscribed within a circle of radius 6 inches. Find the area of the circle, of the hexagon, and of each of the segments of the circle subtended by the sides of the hexagon.

3. In a triangle with base 12 inches and altitude 6 inches, a line is drawn parallel to the base, cutting off a triangle whose area is one-half the area of the given triangle. Find the base and altitude of the smaller triangle.

4. (*a*) What is the locus of all points in space equidistant from two fixed points?

(*b*) What is the locus of all points in space equidistant from two intersecting planes?

5. A brick chimney is to be built in the form of a hollow right circular cylinder, 1 foot thick, 8 feet in outer diameter, and 30 feet high. Find the cost of construction at $2 per cubic foot.

6. A barn is 25 feet wide, 45 feet long, and 12 feet high from the ground to the eaves; the sloping sides of the roof meet at right angles. Find the total capacity of the barn.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO WORCESTER POLYTECHNIC INSTITUTE

JUNE, 1909

9 to 10.30 A.M.

(NOTE. *Give reasons in full for any statement made in the course of a demonstration.*)

1. Define:
- (*a*) A regular polygon.
- (*b*) A rhombus.
- (*c*) A rhomboid.
- (*d*) An apothem of a polygon.
- (*e*) A secant of a circle.

2. Demonstrate: The medians of a triangle meet in a point which is two-thirds of the distance from each vertex to the middle of the opposite side.

3 Demonstrate: The tangents to a circle drawn from an exterior point are equal and make equal angles with the line joining the point to the centre.

4. Find the area of an equilateral triangle in terms of its side. Denote the side by a and the area by S.

5. The chord of an arc is 24 inches and the height of the arc 9 inches. Find the diameter of the circle.

SEPTEMBER, 1909

9 to 10.30 A.M.

(NOTE. *Give reasons in full for any statement made in the course of a demonstration.*)

1. Define:
  (*a*) A median of a triangle.
  (*b*) A segment of a circle.
  (*c*) A radius of a polygon.
  (*d*) The locus of a point.
  (*e*) A tangent to a circle.

2. Demonstrate: The areas of two similar segments of circles are to each other as the squares of their radii.

3. Compute the altitude H of a triangle in terms of its sides A, B and C.

4. Demonstrate: The square of the bisector of an angle of a triangle is equal to the product of the sides of this angle diminished by the product of the segments determined by the bisector upon the third side of the triangle.

5. The chord of half an arc of a circle is 12 feet and the radius of the circle is 18 feet. Find the height of the arc.

JUNE, 1910

(Time allowed, one hour and a half.)

(NOTE. *Give reasons in full for any statement made in the course of a demonstration.*)

1. Define:
  (*a*) A trapezoid.
  (*b*) A trapezium.
  (*c*) The median of a trapezoid.
  (*d*) A common interior tangent to two circles.
  (*e*) A sector of a circle.

2. Demonstrate: If a figure is symmetrical with respect to two axes perpendicular to each other, it is symmetrical with respect to their intersection as a centre.

3. Demonstrate: In the same circle or equal circles equal chords are equally distant from the centre; and conversely if the chords are equally distant from the centre they are equal.

4. Demonstrate: The bisector of an exterior angle of a triangle meets the opposite side produced at a point the distances of which from the extremities of this side are proportional to the other two sides.

5. The radii of two circles are 8 inches and 3 inches, and the distance between centres is 15 inches. Find the lengths of their common tangents.

SEPTEMBER, 1910

9 to 10.30 A. M.

1. Define:
- (*a*) An isosceles trapezoid.
- (*b*) A common exterior tangent to two circles.
- (*c*) An apothem of a polygon.
- (*d*) A re-entrant angle of a polygon.
- (*e*) A segment of a circle.

2. Demonstrate: An angle formed by a tangent and a chord is measured by one-half the intercepted arc.

3. Demonstrate: If a straight line divide two sides of a triangle proportionally, it is parallel to the third side.

4. Construct: A parallelogram equivalent to a given square, and having the difference of its base and altitude equal to a given line.

5. From the end of a tangent 20 inches long a secant is drawn through the centre of the circle. If the exterior segment of this secant is 8 inches, find the radius of the circle.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO PRINCETON

JUNE, 1903

Omit one question.

1. The bisectors of the angles of any triangle meet in a point.

2. If through a point of a circle a tangent and a chord be drawn, the angle between these lines is measured by half the intercepted arc.

3. Two triangles having an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles.

4. The sides of a triangle are 5, 5, and 8 meters respectively; find the radius of a circle whose area is equal to that of the triangle.

5. The side of a rhombus is 52 meters and one of its diagonals is 96 meters; find its area.

6. In a quadrilateral the diagonals are equal and also one pair of opposite sides is equal. Prove that two of the triangles into which the quadrilateral is divided by the diagonals are isosceles.

7. The common points A and B of two circles are joined to a point P on one of the circles and the lines PA and PB cut the other circles again in C and D respectively. Prove that CD is parallel to the tangent at P.

8. The bisectors of the exterior angles at B and C of a triangle ABC meet in a point D. Show that twice the angle BDC is the supplement of the angle A.

JUNE, 1904

Omit one question.

1. The sum of two lines drawn from a point to the extremities of a straight line is greater than the sum of two other lines similarly drawn but included by them.

2. The perpendicular erected at the extremity of a radius of a circle is tangent to the circle.

3. In any obtuse-angled triangle the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides plus twice the product of one of these sides and the projection of the other side upon it.

4. If the diagonal AC of a quadrilateral $ABCD$ divides it into two equivalent triangles, AC bisects BD.

5. If a hexagon has its opposite sides equal and parallel, the three lines joining the opposite angles meet in a point.

6. In the triangle ABC, D is the foot of the perpendicular from A upon the base BC, and E and F are the middle points of the sides AB and AC respectively; prove that the angle EDF is equal to the angle EAF and that the area of the quadrilateral $EDFA$ is equal to one-half that of the triangle ABC.

7. Find the radius of the circle whose area is equal to that of a regular hexagon whose perimeter is 36.

8. The sides of a triangle are 3, 6, 7; find the perimeter of a similar triangle whose area is three times as great as that of the given triangle.

JUNE, 1905

Omit one question.

1. If two circles touch each other internally, the straight line which joins their centres being produced will pass through the point of contact.

2. In any right triangle the square described upon the hypotenuse is equivalent to the sum of the squares described upon the legs.

3. The straight line perpendicular to a radius of a circle at its extremity meets the circle in but one point; and if a straight line meet a circle in but one point, it is perpendicular to the radius at that point.

4. The four triangles into which a parallelogram is divided by its diagonals are equal in area.

5. Given any eight-sided polygon $ABCDEFGH$ inscribed in a circle, prove that the sum of the angles A, C, E, and G is equal to six right angles.

6. Given a triangle ABC and a fixed point D on AC; find the line through D which divides the triangle into two parts of equal area.

7. The sides of a triangle are 5, 12, 13; find the radius of the circle whose area is equal to that of the triangle.

8. In a triangle ABC the angle C is a right angle and the lengths of AC and BC are 5 and 12 respectively; the hypotenuse BA is produced through A to a point D so that the length of AD is 4; CA is produced through A to E so that the triangles AED and ABC have equal area; what is the length of AE?

JUNE, 1906

1. Prove that in any triangle having an obtuse angle the square of the sides opposite the obtuse angle is equal to the sum of the squares of the other two sides, plus twice the product of one of these sides and the projection of the other side upon it.

2. Prove that the lines joining the vertices of a triangle to the mid-points of the opposite sides have one point in common.

3. Prove that a line which divides two sides of a triangle proportionally is parallel to the third side.

4. Find the locus of the point at which a given segment of a straight line subtends a given constant angle.

5. A quadrilateral is formed by the diameter *AB* of a circle, the two tangents at *A* and *B*, and a third tangent which meets the other tangents in *C* and *D* respectively; prove that the area is equal to one-half the product of the opposite sides *AB* and *CD*.

6. A circle of radius 4 has its center at the intersection of the diagonals of a square whose side is 12; find the length of the circumference of a circle which touches two adjacent sides of the square and also the circle.

JUNE, 1907

1. Prove that any point which is equidistant from two given intersecting straight lines lies on the bisector of one of the angles between the given lines.

2. An angle at the centre of a circle is double an angle at the circumference which stands on the same arc.

3. In any triangle, the product of any two sides is equal to the diameter of the circumscribed circle, multiplied by the perpendicular drawn to the third side from the vertex of the opposite angle.

4. The line *AB* is a diameter and the point *C* is the centre of a given circle *ADB*. Another circle is drawn touching the circumference *ADB* at *D* and the diameter *AB* at *E*. Prove that *O*, the centre of this circle, is equidistant from *C* and one of the tangents to the given circle which is parallel to *AB*.

5. Prove that the middle points of the sides of any quadrilateral are the vertices of a parallelogram. If the area of the original quadrilateral is 60 square feet what is the area of this parallelogram?

6. Find the area of the segment of a circle of radius 4 cut off by a chord which subtends a right angle at the centre of the circle.

JUNE, 1908

1. The diagonals of a parallelogram bisect each other.

2. Show how, upon a given straight line, to describe a segment of a circle which shall contain an angle of given magnitude.

3. If C be the middle point of AB and CD be the line through C perpendicular to AB, show that every point of CD is equidistant from A and B and that points on the side of CD on which A lies are nearer to A than to B.

4. A circle $ABCD$ is cut by a second circle in the points A and B and by a third circle in C and D. The lines AB and CD meet in a point O. Prove that all the tangents which can be drawn to the circles from O are equal in length.

5. The triangle whose vertices are respectively one of the vertices of a parallelogram and the middle points of the sides of the parallelogram remote from this vertex is equivalent to three-eighths of the parallelogram.

SEPTEMBER, 1908

1. The sum of the three interior angles of a triangle is equal to two right angles.

2. If two chords of a circle intersect, the product of the segments of the one is equal to the product of the segments of the other.

3. The areas of similar triangles are to each other as the squares of their homologous sides.

4. The diameter AB of a circle is trisected at the points C, D. On AC and AD as diameters, semicircumferences are described on one side of AB; and on DB and CB as diameters, semicircumferences are described on the other side of AB. Prove that the circle is thereby divided into three equivalent parts.

5. What is the ratio of the areas of two circles, one inscribed in, and the other circumscribed about, an equilateral triangle?

JUNE, 1909

1. In any triangle the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides minus twice the product of one of these sides and the projection of the other side upon it.

2. If any two chords be drawn through a fixed point within a circle the product of the segments of one chord is equal to the product of the segments of the other.

3. In any triangle the bisector of an exterior angle divides the opposite side externally into segments proportional to the adjacent sides.

4. The two sides of a triangle are together greater than twice the straight line drawn from the vertex to the middle point of the base.

5. AB is the common chord of two circles; through C any point of one circumference, straight lines CAD and CBE are drawn terminated by the other circumference. Show that the length of DE will be the same no matter what the position of C.

6. Find the radius of the inscribed circle and that of the circumscribed circle to an equilateral triangle whose side is 6.

SEPTEMBER, 1909

1. If two triangles have two sides of one equal respectively to two sides of the other but the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second; and conversely if the third side of the first is greater than the third side of the second, then the included angle of the first is greater than the included angle of the second.

2. The line joining the middle points of the non-parallel sides of a trapezoid is parallel to the bases and equal to one-half their sum.

3. The areas of two similar triangles are proportional to the squares of two homologous sides.

4. If the angle contained by any side of a quadrilateral and the adjacent side produced be equal to the opposite angle of the quadrilateral, show that any side of the quadrilateral will subtend equal angles at the opposite vertices of the quadrilateral.

5. Tangents AB and AC are drawn to a circle touching it at the points B and C; the diameter through B meets the circle again at D; prove that CD is parallel to the line joining A to the centre of the circle.

6. Find the area of the square inscribed in a semicircle whose radius is 10.

JUNE, 1910

1. The line joining the middle points of two sides of a triangle is parallel to the third side and equal to one-half of it.

2. In a triangle the product of any two sides is equal to the perpendicular from their intersection to the third side multiplied by the diameter of the circumscribed circle.

3. The perpendiculars from the vertices of a triangle to the opposite sides meet in a point.

4. From the extremities of the base of an isosceles triangle, straight lines are drawn perpendicular to the sides; show that the angles made by them with the base are each equal to half the angle at the vertex.

5. The sides AB and AC of a given triangle ABC, are bisected at the points E and F; a perpendicular is drawn from A to the opposite side BC, meeting it at D. Show that the angle FDE is equal to the angle BAC. Show also that $AFDE$ is half the triangle ABC.

6. Two circles cut at A and B. Through A and B the parallel straight lines CAD and EBF are drawn, meeting the circles in C, D, E, F. Prove that CD is equal to EF.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO THE UNIVERSITY OF PENNSYLVANIA

JUNE, 1906

(Time, two hours.)

(*Write legibly your full name, and the course for which you are applying, on the cover of each book used.*)

1. Define: Parallel lines, equivalent figures, segment of a circle. What is a postulate? Give an example.

2. Prove that two triangles are equal when the three sides of one are equal respectively to the three sides of the other.

3. In the same circle, or in equal circles, two angles at the centre are in the same ratio as their intercepted arcs.

4. Define similar polygons, and prove that if two polygons be composed of the same number of triangles, similar each to each and similarly placed, the polygons are similar.

5. Similar triangles are to each other as the squares of their homologous sides.

6. Find the locus of a circumference which passes through two given points.

7. The lines joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram. The perimeter of the parallelogram so formed is equal to the sum of the diagonals of the quadrilateral.

8. Find the area of an equilateral triangle whose altitude is $3\sqrt{3}$.

JUNE, 1908

(Time, two hours.)

1. Define: Median of a triangle, chord of a circle, similar polygons, apothem of a regular polygon, sector of a circle.

2. The base and altitude of a parallelogram are 10 and 15 inches, respectively. Compute the length of the side of an equivalent equilateral triangle.

3. If three or more parallels intercept equal lengths on one transversal, they intercept equal lengths on every transversal.

4. An angle formed by two secants to a circle is measured by half the difference between the intercepted arcs.

5. To a given circle draw a tangent parallel to a given line.

6. In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

7. Two triangles are similar if they are mutually equiangular.

8. Regular polygons of the same number of sides are similar.

SEPTEMBER, 1908

(Time, two hours.)

1. Define: Rhombus, secant to a circle, segment of a circle, regular polygon, projection of one line upon another.

2. The hypotenuse of a right triangle is 20, and the projection of one of the sides upon the hypotenuse is 4. What is the area of the triangle?

3. If two sides of a triangle are unequal, the opposite angles are unequal, and the greater angle is opposite the greater side.

4. The tangents drawn to a circle from a point without are equal.

5. Construct the length $a\sqrt{5}$, if the length a is given.

6. Two triangles are similar, if an angle of the one is equal to an angle of the other, and the sides including this angle are proportional.

7. Similar triangles are to each other as the squares of their homologous sides.

8. The perimeters of regular polygons of the same number of sides have the same ratio as their radii.

SEPTEMBER, 1909

(Time, two hours.)

1. Define: perpendicular lines, distance from a point to a line, sector of a circle, the mean proportional between two magnitudes, equivalent figures.

2. The line that joins the vertices of two isosceles triangles on the same base bisects the common base at right angles.

3. In similar triangles bisectors of homologous angles have the same ratio as any two homologous sides.

4. Transform any triangle ABC into an equivalent isosceles triangle having its base equal to AB.

5. Construct a rectangle equivalent to a given square, having the sum of its base and altitude equal to a given line.

6. The area of a circle is equal to one-half the product of its circumference and radius.

7. The sides of a triangle are $AB = 13$, $BC = 14$, $CA = 15$. Find the projection of AB upon BC.

8. An angle formed by a tangent to a circle and a chord drawn from the point of contact is measured by half the intercepted arc. Explain what is meant by the expression " measured by " as used in the statement of this proposition.

SEPTEMBER, 1910

(Time, two hours.)

1. Define or explain the terms: quadrilateral, mean proportional, sector of a circle, rhombus, parallel lines.

2. The bisector of an angle is the locus of points within the angle and equally distant from the sides of the angle.

3. Two parallels intercept equal arcs on a circumference.

4. The angle formed by two secants intersecting without a circle is measured by half the difference of the intercepted arcs.

5. Two triangles having an angle of one equal to an angle of the other are to each other as the products of the sides including the equal angles.

6. Compute the area of a square inscribed in a circle whose circumference is $157\frac{1}{7}$ feet (take $\pi = \frac{22}{7}$).

7. Construct a square equal to a given parallelogram and prove the correctness of the construction.

8. The lengths of two sides of a triangle are 7 and 9, and the median drawn to the third side equals 7. Find the length of the third side.

EXAMINATIONS IN GEOMETRY FOR ADMISSION TO COLUMBIA UNIVERSITY

SEPTEMBER, 1908

PLANE AND SOLID GEOMETRY

(NOTE. *Time: Three hours. Candidates offering only plane geometry are to answer all the questions of group A and two from each of the groups B, C. Those offering only solid geometry are to answer all the questions of group D and two from each of the groups E, F. Those offering both subjects are to answer two questions from each of the groups B, C, E, F.*)

On the cover of his examination book each candidate will state what text-book or text-books he has used in preparation for this examination, and will indicate whether his examination is in plane geometry, solid geometry, or plane and solid geometry.

A

1. Prove that two triangles are equal if two angles and the included side of one are equal respectively to the two angles and included side of the other.

2. Prove that if three or more parallel straight lines cut off equal segments on one transversal, they cut off equal segments on every transversal.

3. Show how to circumscribe a circle about a given triangle. Prove the correctness of your method.

B

4. Prove that two parallel chords intercept equal arcs on the circumference of a circle.

5. In a right triangle, a perpendicular is drawn from the vertex of the right angle to the hypotenuse.

(*a*) Prove that either leg is a mean proportional between the whole hypotenuse and the segment adjacent to that leg.

(*b*) If the hypotenuse is 10, and the perpendicular from the vertex of the right angle is 4, into what segments does the perpendicular divide the hypotenuse?

6. Prove that the areas of any two similar triangles have the same ratio as the squares of their corresponding sides.

C

7. Prove that an angle inscribed in a circle is measured by one-half the arc intercepted between its sides.

8. Find the locus of the vertex of a triangle which has a fixed base and a given area.

9. The area of a circle is 36π. Find the side, apothem, and area of the inscribed equilateral triangle.

D

10. Prove that of all straight lines drawn from a given point to a given plane,

(*a*) The perpendicular is the shortest line;

(*b*) Any two oblique lines which cut off unequal distances from the foot of the perpendicular are unequal, the one which cuts off the greater distance being the greater.

11. Prove that the sum of any two face angles of a triedral angle is greater than the third.

12. Prove that if a pyramid is cut by a plane parallel to the base, the edges and altitude are divided proportionally, and the section is a polygon similar to the base.

E

13. Prove that the locus of points in space which are equidistant from two given fixed points is the plane which bisects at right angles the straight line joining the given points.

14. Prove that the plane passed through two diagonally opposite edges of a parallelopiped divides the parallelopiped into two equivalent triangular prisms.

15. Prove that the area generated by the revolution of a straight line about an axis coplanar with it, but not parallel to it, is equal to the projection of the line on the axis multiplied by the circumference of the circle whose radius is the perpendicular erected at the mid-point of the line, and terminated in the axis.

F

16. A right circular cone whose altitude is 12 feet, and the radius of whose base is 9 feet, is cut by a plane 4 feet from and parallel to the base; find the lateral surface and the volume of the frustum thus formed.

17. A cylinder is circumscribed about a sphere of radius R; find the ratio of their surfaces and also the ratio of their volumes.

18. Prove that if one spherical triangle is the polar of a second spherical triangle, then the second is also the polar of the first.

SEPTEMBER, 1909

PLANE AND SOLID GEOMETRY.

(NOTE. *Time: Three hours. Candidates offering only plane geometry are to answer all the questions of group A and two from each of the groups B, C. Those offering only solid geometry are to answer all the questions of group D and two from each of the groups E, F. Those offering both subjects are to answer two questions from each of the groups B, C, E, F.*)

On the cover of his examination book each candidate will state what text-book or text-books he has used in preparation for this examination, and will indicate whether his examination is in plane geometry, solid geometry, or plane and solid geometry.

A

1. (*a*) Show by a diagram that two triangles may be unequal, even though they have two sides and an angle of one equal respectively to two sides and an angle of the other.

(*b*) Prove that two triangles are equal if two sides and the *included* angle of one are equal respectively to two sides and an included angle of the other.

2. Prove that the bisector of an angle is the locus of points equidistant from the sides of the angle. How would you find a point which is five units distant from each side of the angle?

3. Prove that in the same circle or in equal circles, the less of two chords is at the greater distance from the centre.

B

4. Show how to construct a tangent to a given circle through a given point,

(*a*) when the given point is on the circle;

(*b*) when the given point is outside the circle.

5. Prove that the bisector of an exterior angle of a triangle divides the opposite side externally into segments proportional to the other two sides of the triangle.

6. The legs of a right-triangle are 5 and 12; find the length of the perpendicular from the vertex of the right angle to the hypotenuse, and the lengths of the segments into which the perpendicular divides the hypotenuse.

C

7. Prove that the areas of any two similar polygons are to each other as the squares of any two corresponding sides.

8. Prove that if two equal chords of a circle are produced to meet outside the circle, the exterior parts are equal.

9. Prove that if the mid-points of two adjacent sides of a parallelogram are joined by a straight line, the area of the triangle formed is equal to one-eighth of the area of the parallelogram.

D

10. Prove that if two straight lines are cut by three parallel planes, the lines are divided proportionally.

11. Prove that if two planes are perpendicular to each other, a straight line drawn in one plane, perpendicular to the intersection of the two planes, is perpendicular to the other plane.

12. Prove that the sum of the face angles of any convex polyhedral angle is less than four right angles.

E

13. Prove that if a pyramid is cut by a plane parallel to the base,

(*a*) the section is a polygon similar to the base;

(*b*) the areas of the section and the base are to each other as the squares of their distances from the vertex.

14. Prove that the locus of points in space equidistant from two given points is the plane bisecting at right angles the straight line joining the given points.

15. Prove that the area generated by a straight line revolving about a non-parallel straight line coplanar with it is equal to the length of the generating line, multiplied by the circumference generated by its mid-point.

F

16. Prove that if two triangles on the same, or on equal, spheres have the three angles of one equal respectively to the three angles of the other, then the three sides of the one are equal respectively to the three sides of the other.

17. The radius of the base of a right circular cone is 3; the radius of the circumscribed sphere is 5. Find the ratio of the volumes of the sphere and cone.

18. Find the area, in square feet, of the spherical quadrilateral whose angles are 100°, 120°, 140°, and 160°, the radius of the sphere being 10½ feet.

SEPTEMBER, 1910

PLANE AND SOLID GEOMETRY.

(NOTE. *Time: Three hours. This paper will be rated as a whole; separate credits will not be given on this paper for plane and solid geometry. Eight questions are required, four from Group A, and four from Group B. No credit will be given for more than eight questions.*)

On the cover of his examination book each candidate will state what text-book or text-books he has used in preparation for this examination, and will indicate whether his examination is in plane geometry, solid geometry, or plane and solid geometry.

A

1. Prove that if three or more parallel lines intercept equal segments on one transversal, they intercept equal segments on every transversal.

2. Prove that if from a point without a circle a secant and a tangent are drawn, the tangent is a mean proportional between the secant and its external segment.

3. Prove that the areas of two triangles which have an angle of the one equal to an angle of the other are to each other as the products of the sides enclosing the equal angles.

4. If one side of an equilateral triangle is *a*, find the altitude and area of the triangle, and the radii of the circumscribed and inscribed circles.

5. The point *P* moves so that the two tangents from *P* to two given intersecting circles are equal; prove that the locus of *P* is the produced common chord of the given circles.

B

6. (*a*) Define parallel lines.

(*b*) Prove that if two parallel planes are cut by a third plane, the lines of intersection are parallel.

7. Prove that if two trihedral angles have the face angles of the one equal respectively to the face angles of the other, then the dihedral angles of the one are equal respectively to the dihedral angles of the other.

8. Prove

(*a*) that parallel sections of a prism are equal polygons.

(*b*) that parallel sections of a pyramid are similar.

9. Complete and prove the following theorem: The surface generated by a straight line revolving about an axis in the same plane with it is equal to the product of the projection of the line upon the axis by the circumference of the circle whose radius is ——.

10. A cylinder whose altitude is 12 is inscribed in a sphere whose radius is 10. Find the volume and surface of the sphere, the volume and lateral surface of the cylinder, the area of the zone included between the bases of the cylinder, and the area of the spherical triangle whose angles are 138°, 122°, and 55°.

SEPTEMBER, 1910

Solid Geometry

(Note. *Time: Three hours. Answer seven questions. No credit will be given for more than seven questions.*)

On the cover of his examination book each candidate will state what text-book or text-books he has used in preparation for this examination, and will indicate whether his examination is in plane geometry, solid geometry, or plane and solid geometry.

1. (*a*) Define parallel lines.

(*b*) Prove that if two parallel planes are cut by a third plane, the lines of intersection are parallel.

2. Prove that the plane which bisects a dihedral angle is the locus of points equidistant from the faces of the angle.

3. Prove that if two trihedral angles have the face angles of the one equal respectively to the face angles of the other, then the dihedral angles of the one are equal respectively to the dihedral angles of the other.

4. Prove

(*a*) that parallel sections of a prism are equal polygons.

(*b*) that parallel sections of a pyramid are similar.

5. Prove that two pyramids having equivalent bases and equal altitudes are equal in volume.

6. The surface generated by a straight line revolving about an axis in the same plane with it is equal to the product of the projection of the line on the axis by the circumference of the circle whose radius is ——. Complete and prove this theorem.

7. Prove that the sum of the three sides of a spherical triangle is less than 360°, and that the sum of the three angles is greater than 180°.

8. A cylinder whose altitude is 12 is inscribed in a sphere whose radius is 10. Find the volume and surface of the sphere, the volume and lateral surface of the cylinder, the area of the zone included between the bases of the cylinder, and the area of the spherical triangle whose angles are 138°, 122°, and 55°.

SEPTEMBER, 1910

PLANE GEOMETRY.

(NOTE. *Time: Three hours. Answer six questions from Group A, and one from Group B. No credit will be given for more than seven questions.*)

On the cover of his examination book each candidate will state what text-book or text-books he has used in preparation for this examination, and will indicate whether his examination is in plane geometry, solid geometry, or plane and solid geometry.

A

1. Prove that if two sides of a triangle are unequal, the angles opposite are unequal, and the greater angle is opposite the greater side.

2. Prove that if three or more parallel lines cut off equal segments on one transversal, they cut off equal segments on every transversal.

3. Prove that the common chord of two intersecting circles is perpendicular to their line of centres.

The common chord of two circles is 24 feet in length; the radii of the circles are 13 feet and 20 feet respectively. Find the distance between the centres of the circles.

4. Prove that if from a point without a circle a secant and a tangent are drawn, the tangent is a mean proportional between the whole secant and its external segment.

5. If one side of an equilateral triangle is a, find the altitude and area of the triangle, and the radii of the circumscribed and inscribed circles.

6. Prove that in any plane triangle, the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides minus twice the product of one of these sides by the projection of the other side upon it.

Show what this theorem becomes when one of these latter two sides is perpendicular to the other.

7. Prove that the areas of two triangles which have an angle of the one equal to an angle of the other are to each other as the products of the sides enclosing the equal angles.

B

8. The lines joining successively the mid-points of the sides of any quadrilateral enclose a parallelogram.

Show that the area of this parallelogram is equal to half the area of the given quadrilateral.

9. The area of a 10-degree sector of a certain circle is equal to the area of the ring between two concentric circles whose radii are 15 feet and 12 feet. Find the radius of the first-named circle.

10. The point P moves so that the two tangents from P to two given intersecting circles are equal; prove that the locus of P is the straight line through the points of intersection of the given circles.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO CORNELL

1902

1. The perpendicular is the shortest line from a given point to a given straight line.

2. Construct a square equivalent to the difference of two given squares, and a triangle equivalent to this square.

3. In any right triangle the perpendicular from the vertex of the right angle to the hypotenuse is a mean proportional between the segments of the hypotenuse.

4. The radius of a circle being 20 feet, compute the area of a segment whose arc is 300°.

5. The areas of two similar triangles are to each other as the squares of two homologous sides.

6. The circumference of two circles are to each other as their radii, and their areas as the squares of their radii.

7. If from the diagonal BD of a square $ABCD$, BE be cut off equal to BC, and EF be drawn perpendicular to BD, then $DE = EF = FC$.

1904

1. Prove that the area of a trapezoid is double the area of a triangle whose base is one of the non-parallel sides of the trapezoid, and whose vertex is the middle point of the opposite side.

2. Define a locus: Find the locus of the midpoints of all the lines drawn from a given point to a given line not passing through the point.

3. Inscribe a regular decagon in a given circle.

4. The base of an isosceles triangle is a, and the perpendicular let fall from an extremity of the base to the opposite side is b. Find the lengths of the equal sides.

5. Divide a given line so that the rectangle of the two segments may be equivalent to a given square.

6. In a right triangle any polygon standing on the hypotenuse is equivalent to the sum of two similar and similarly situated polygons standing on the other two sides. Prove.

7. Construct a polygon similar to a given polygon and such that the ratio of similitude shall be equal to a given ratio.

8. The side of an equilateral triangle is 1 meter, compute the area of its inscribed circle.

1905

One question may be omitted.

1. Define: adjacent angles; right angle; projection of a line-segment; third proportional; contact of two circles.

2. Two chords of a circle intersect within the circle; prove that the rectangle of the segments of one chord is equivalent to the rectangle of the segments of the other.

3. Show how to construct a polygon similar to one given polygon, and equivalent to another.

4. The base of a triangle is given in magnitude and position and the difference of the squares on the other two sides is also given; find the locus of the vertex.

5. For a regular hexagon, compute the ratio of the areas of the inscribed and the circumscribed circles.

6. Prove that any side of a triangle is less than the sum of the other two, and greater than their difference.

7. Given two sides of a triangle, construct it so that the area may be a maximum.

8. The perimeter of a square is b meters, compute the perimeters of an equilateral triangle and of a circle, each having the same area as the square.

1906

1. Define: parallel lines, equal arcs, supplementary angles, incommensurable magnitudes, similarly divided lines.

2. Prove that the sum of the three angles of a triangle is equal to two right angles.

3. Complete and prove the following statement:

The sum of the squares on two sides of a triangle exceeds twice the square on half the third side by . . .

4. Prove that the areas of two circles are to each other as the squares on their radii.

5. Construct a triangle, being given the base, area, and vertical angle.

6. Find the locus of a point whose distances from two given points are in the ratio of m to n.

7. At what distance from the centre of a circle of radius r is the circle seen to subtend an angle of 60°? Prove.

1908

1. Define: the converse of a proposition, a straight line, the complement of an angle, a regular polygon, a square, similar figures.

2. Prove that if from a given point within a triangle straight lines be drawn to the extremities of any side of the triangle, the sum of these two lines is less than the sum of the other two sides of the triangle.

3. Prove that the sum of all the angles of a convex polygon is the product of two right angles by the number of sides less two.

How many sides has that polygon the sum of whose angles is sixteen right angles.

4. A diameter that is perpendicular to a chord bisects the chord and the subtended arc.

5. If from a point without a circle two straight lines be drawn to cut it, the rectangle of one secant and its external segment is equal to the rectangle of the other secant and its external segment.

6. Two rectangles that have the same altitude are to each other as their bases. (Two cases.)

7. A circle is equal in area to half the rectangle of the radius and circumference.

8. How many degrees are there in an angle whose bounding arc is as long as the radius.

1909

1. Prove that the bisector of an interior angle of a triangle divides the opposite side into segments that are proportional to the other two sides.

2. Derive a formula for the area of a trapezoid. Find the area of the trapezoid $ABCD$, being given $AB = 10$ cm., $BC = DA = 4$ cm., and $CD = 5$ cm.

3. What is the complete locus of points equidistant from two intersecting straight lines? Prove that your answer is correct.

4. From a given point A tangents are drawn to all the circles with centres at another point B and with radii not greater than BA. Find the locus of the points of contact.

5. Show how to construct a circle that shall pass through two given points and be tangent to a given line. Prove that your construction is correct. How many solutions are there in general?

6. Prove that the areas of two similar triangles are to each other as the squares of two homologous sides of the triangles.

7. Find the area of the segment of a circle of radius 10 inches that is cut off by a chord whose length equals the radius.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO DARTMOUTH

JUNE, 1907

(*Take eight numbers—omit one of first five and one of second five.*)

1. Three lines intersect in a point, two angles are 60° and 40°, respectively; find the number of degrees in each of the other four angles. Prove that the sum of the angles of any plane triangle is 180°.

2. Prove that two parallel secants intercept equal arcs.

In a circle whose radius is 13, two chords are 5 and 12, respectively, from the centre; compare their lengths.

3. Prove that, when through a point within a circle any chord is drawn, the product of the two segments is the same whatever its direction.

Two chords intersect within a circle, the segments of one are each 4, the total length of the other is 10; find the length of its segments.

4. State and prove the relation between the lengths of the three sides of a right-angled triangle. The sides of a triangle are 4, 12, 13; is it right, acute, or obtuse angled?

5. The sides of an equilateral triangle are each 9, two are trisected by lines parallel to the base; compare the perimeters and areas of the three parts.

6. Find the locus of a point which is 5 from a given point, also of a point which is 5 from each of two given points which are 6 apart.

7. How many diagonals has a polygon of n sides? Find one angle of a regular polygon of 60 sides.

8. The radius of a circle is 6, a point is 10 from its centre: find the lengths of the tangents drawn from this point to the circumference.

9. One side of an equilateral triangle is $6\sqrt{3}$; find the radius of the inscribed circle, also of the circumscribed circle.

10. The radius of a circle is 10; find the perimeter and area of the part included between an arc of 60° and its chord. Name this part.

JUNE, 1908

(Take eight numbers—omit one of first five and one of last five.)

1. To draw a line through a given point and perpendicular to a given line (*a*) when the point is on the line, (*b*) when off the line.

2. State the theorems for the equality of triangles. What three mutually equal parts may two unequal triangles have?

3. Find one angle of a regular polygon of 6 sides, 12 sides, 15 sides. What equal regular polygons will exactly cover a plane surface?

4. In any triangle, the bisector of an angle divides the opposite side into segments proportional to the adjacent side. Prove.

5. In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Prove.

6. The angle between two intersecting lines which cut a circle is 40°, one of the intercepted arcs is 60°. Find the other intercepted arc (*a*) when the point of intersection is within the circle, (*b*) when it is outside the circle.

7. Find the locus of a point which is equally distant from (*a*) two given points, (*b*) two intersecting lines, (*c*) the three sides of a triangle.

8. A tangent and a secant are drawn from a point outside a circle; the square of the tangent equals the product of the whole secant and its external segment. Prove.

9. The sides of a triangle are 5, 5, 6; find its area and the area of its inscribed circle.

10. The radius of a circle is 6. Find the area of the inscribed regular polygon of 4 sides; of 12 sides.

SEPTEMBER, 1909

(*Take eight numbers—four of the first five and four of the last five.*)

1. Define and classify lines, angles, and triangles.

2. Prove that two triangles are equal when they have three consecutive parts equal...

3. Find one angle of regular polygons of 10, 20, and 40 sides. What is the largest angle a regular polygon can have?

4. The sides of a triangle are 4, 5, 6 in lengths. The sides 4 and 6 are produced beyond their intersection and the pairs of vertical angles are bisected. Find where these bisectors meet the base and the base produced.

5. From a point outside a circle a secant is drawn through the centre. What lines represent the arithmetic and geometric means between the whole secant and the external segment?

6. Prove that an angle included between a secant and a tangent meeting at the point of tangency is measured by half the intercepted arc.

7. Two chords are 5 and 12, respectively, from the centre of a circle whose radius is 13: compare their lengths.

8. Two chords intersect, prove that the product of the segments of one equals the product of the segments of the other, and state the general proposition.

9. The sides of an equilateral triangle are a, find its altitude and area.

10. The radius of a circle is 10; find the perimeter and area of a segment cut off by a chord which subtends an arc of 60°.

SEPTEMBER, 1910

(*Take eight numbers—four of the first five and four of the last five.*)

1. Define the classes of quadrilaterals. Prove that the diagonals of a rhombus bisect each other at right angles.

2. How many diagonals has a polygon of n sides? Prove that the sum of its angles is $(n-2)$ 180°.

3. Inscribe a circle in a triangle and state the propositions involved.

4. Show how to draw the two tangents to a circle from a point outside and prove that they are equal.

5. The sides of a triangle are 36, 42, 48; find the segments of the sides made by the bisectors of the angles.

6. Prove that parallel chords intercept equal arcs. In a circle whose radius is 9, two chords are 6 and 7 from the centre. Compare their lengths.

7. Prove that the medians of a triangle mutually trisect each other.

8. The base of a triangle is 7, the other sides are 5 and 6. Find where the altitude meets the base.

9. The sides of a triangle are each $4\sqrt{3}$. Compare the areas of the inscribed and circumscribed circles.

10. In a circle whose radius is 6 find the perimeter and area of a segment whose arc is 90°.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO BROWN

SEPTEMBER, 1909

1. Through three points not lying in a straight line not more than one circle can be drawn.

2. An angle formed by a tangent and a chord is measured by one-half the intercepted arc.

3. Two circles are tangent internally at A. Draw two chords AB and AC to the exterior circle, cutting the interior circle at D and E. Draw BC and DE. Prove BC and DE are parallel lines.

4. AB is tangent to a circle at B. AD is a secant line to the same circle cutting the circumference at C and D. $AD=60$ inches and $AB=40$ inches. What is the length of the chord CD?

5. The area of a trapezoid is one-half the product of its altitude and the sum of its parallel sides.

6. The two sides of a right triangle are respectively 24 and 10 inches long, and a perpendicular is dropped from the right angle upon the hypotenuse. Find the length of the hypotenuse, of each of its segments, and of the perpendiculars upon it.

SEPTEMBER, 1910

1. Of two oblique lines drawn from the same point in a perpendicular and cutting off unequal distances from the foot of the perpendicular the more remote is the greater.

2. Two triangles are congruent if the sides of one are equal respectively to the sides of the other.

3. Similar triangles are to each other as the squares of their homologous sides.

4. Problem: To construct a line whose ratio to a given line shall be equal to $\sqrt{3}$.

5. In any obtuse triangle, the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides plus twice the product of one of those sides by the projection of the other upon it.

6. Find the three altitudes and the area of the triangle whose sides are 13, 14 and 15.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO BOWDOIN

JUNE, 1908

4 to 6 P. M.

1. Prove that two triangles are equal when the three sides of one are respectively equal to the three sides of the other.

2. The perpendiculars from the vertices of a triangle upon the opposite sides intersect at a common point.

3. In the same circles or in equal circles two central angles are in the same ratio as their intercepted arcs. Assuming that the theorem has been proved for commensurable arcs, prove it for incommensurable arcs.

4. The angle between two chords intersecting within a circumference is measured by one-half the sum of its intercepted arc and the arc intercepted by its vertical angle.

5. To construct a circle having its centre in a given straight line and passing through two given points, which are not in the line.

6. If through a point without a circle two secants be drawn, the product of the whole secant by its external segment is the same for each secant.

7. Find the perimeter and the area of an equilateral triangle circumscribed about a circle whose area is $17\frac{1}{9}$ sq.in. (take $\pi = \frac{22}{7}$).

JUNE, 1909

4 to 6 P. M.

1. If two triangles have two sides of one respectively equal to two sides of the other, but the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second.

2. If in a right triangle the hypotenuse is twice the shorter leg, then one acute angle is twice the other.

3. The angle between two secants intersecting without the circumference of a circle is measured by one-half the difference of the intercepted arcs.

4. A parallel to one side of a triangle divides the other two sides proportionally. Assuming that the theorem has been proved for commensurable parts, prove it for incommensurable parts.

5. Two similar triangles are to each other as the squares of their homologous sides.

6. To draw a line parallel to the base of a triangle dividing its area into two equivalent parts.

7. With the sides of an equilateral triangle as diameters three semicircumferences are drawn lying wholly without the triangle. Find the perimeter and area of the figure enclosed, if the side of the triangle is 2 inches (take ($\pi = 3.142$.)

JUNE, 1910

4 to 6 P. M.

1. The sum of the angles of any polygon equals twice as many right angles as the figure has sides, less four right angles.

2. In a circle if a chord is parallel to a tangent, the line joining the ends of the chord to the point of contact are equal.

3. Prove the formula for the area of a trapezoid.

4. Construct a triangle similar to a given triangle and of twice its area.

5. Two parallelograms having a common angle are to each other as the products of the sides including this angle.

6. If a perpendicular is drawn from the right angle to the hypotenuse, what three mean proportionals are found in a right triangle? Prove these proportions.

7. A window is in the form of a rectangle surmounted by a semicircle. The area of the opening is 10π and the rectangular part is four-fifths of the whole. What are the dimensions of the window?

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO THE UNIVERSITY OF CHICAGO

SEPTEMBER, 1903

(Time allowed, one hour and thirty minutes.)

(In writing, use only one side of the paper, put your name in full at the top of each sheet, and number your work according to the numbers on the printed paper.)

(When required, give all reasons in full, and work out proofs and problems in detail.)

State:

(*a*) At what school you studied this subject.
(*b*) How many weeks.
(*c*) How many recitations per week.
(*d*) What text-book you used.

1. Show that if straight lines are drawn from any point within a triangle to two of the vertices, the angle contained by these lines is greater than the angle at the third vertex.

2. Prove that any straight line drawn parallel to the base of an isosceles triangle forms with its sides another isosceles triangle.

3. Show how to inscribe a square in a given circle.

4. Prove that the circle described upon the hypotenuse of a right triangle as diameter passes through the vertex of the right angle.

5. A grazier has a movable and flexible fence 440 feet long, with which he encloses his sheep when grazing. What is the difference between the amount of ground enclosed when the fence is arranged in the form of a square and of a circle? Use $\frac{22}{7}$ as π.

DECEMBER, 1904

(Time allowed, one hour and thirty minutes.)

(In writing, use only one side of the paper, put your name in full at the top of each sheet, and number your work according to the numbers on the printed paper.)

(When required, give all reasons in full, and work out proofs and problems in detail.)

State:

(*a*) At what school you studied this subject.
(*b*) How many weeks.
(*c*) How many recitations per week.
(*d*) What text-book you used.

1. Given any angle A and any point P within it; show a method of drawing a straight line to pass through P to the sides of the angle so as to be bisected at P.

2. A boy 5 feet tall casts on the ground a shadow 3 feet long. At the same time of a day a flagpole casts on the ground a shadow 21 feet long. What is the length of the flagpole? Further, what would be the length of a tight rope from tip of the pole to tip of the shadow? Draw a figure of this situation to scale, and verify your results on the figure.

3. Find the locus of all points C such that the angle ABC is a right angle, A and B being two fixed points.

4. Given any triangle ABC, inscribe in it a circle and prove the validity of your construction.

5. What is the area of an equilateral triangle inscribed in a circle of radius 2 feet?

SEPTEMBER, 1905

(Time allowed, one hour and thirty minutes.)

(In writing, use only one side of the paper, put your name in full at the top of each sheet, and number your work according to the numbers on the printed paper.)

(When required, give all reasons in full, and work out proofs and problems in detail.)

State:

(*a*) At what school you studied this subject.
(*b*) How many weeks.
(*c*) How many recitations per week.
(*d*) What text-book you used.

1. Prove that the three perpendicular bisectors of the sides of an obtuse triangle meet in a point.

2. Prove that the sum of the three perpendiculars let fall from any point within an equilateral triangle is equal to the altitude of the triangle.

3. Prove that the square on one side of a triangle equals the sum of the squares on the other two sides, increased or diminished by twice the product of either of those sides and the projection of the other upon it, according as the included angle is obtuse or acute.

4. Through a point 3 inches from the end of a line 10 inches long a perpendicular is erected and a circle is described on the 10-inch line as diameter. Find the length of the chord made by the other line and the distances from its ends to the ends of the diameter.

SEPTEMBER, 1907

(Time allowed, one hour and thirty minutes.)

(In writing, use only one side of the paper, put your name and school at the top of each sheet, and number your work according to the numbers on the printed paper.)

(When required, give all reasons in full, and work out proofs and problems in detail.)

1. If a line is drawn connecting two opposite vertices of a parallelogram, the perpendicular let fall upon this line from the other two opposite vertices are equal. Outline the proof of this theorem and then state and prove separately every theorem quoted.

2. Show how to measure the perpendicular distance AB across a river from a point A by measuring a line AC 100 feet long parallel to the bank, drawing a line CD perpendicular to AC, a distance 50 feet back from the bank and connecting D and B by an imaginary line which cuts AC at a point E. Compute AB, supposing that on measuring AE is found to be 90 feet.

3. State and prove the theorem on similar triangles used in the preceding question.

4. The radius of a circle is 6. Find the difference between the areas of the circle and the inscribed hexagon. Also find an expression representing this difference if the radius of the circle is R.

5. If through one of the points of intersection of two circles the diameters of the circles are drawn, prove that the secant joining the other extremities of these diameters passes through the second intersection point of the circles.

JUNE, 1908

(Time allowed, one hour and thirty minutes.)

(*In writing, use only one side of the paper, put your name and school at the top of each sheet, and number your work according to the numbers on the printed paper.*)

(*When required, give all reasons in full, and work out proofs and problems in detail.*)

State:

(*a*) At what school you studied this subject.
(*b*) How many weeks.
(*c*) How many recitations per week.
(*d*) What text-book you used.

1. Given two circles of unequal radii and lying exterior to each other; make the construction by which a straight line may be drawn tangent to both circles and shall cross the line joining their centres.

2. Prove that the sum of the exterior angles of any convex polygon, made by producing each of its sides in consecutive order, is equal to four right angles.

3. In any triangle PQR perpendiculars are let fall to the opposite sides from the vertices P and Q. Show that the lines joining the feet of these perpendiculars to the middle point of the side PQ are equal. (Draw two figures in one of which the angle at Q shall be obtuse, in the other acute.)

4. Two fields are of similar shape, one having five times the area of the other. (*a*) If they are both circles and the radius of the first is 25 rods, find the radius of the second. (*b*) If they are both equilateral triangles, and the side of the first is 25 rods, find the side of the other.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO BRYN MAWR

AUTUMN, 1908

(Two and one-half hours.)

1. Prove that two straight lines perpendicular to the same straight line are parallel.

2. Prove that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

3. Prove that an angle inscribed in a circle is measured by one-half its intercepted arc.

Three points A, B, C on a circle are joined, and the arcs thus cut off are folded over the lines BC, CA, AB. Prove that when thus folded over these three arcs meet in one point.

4. Give and prove the construction for a circle that shall be inscribed in a given triangle.

Prove that if a quadrilateral inscribed in a circle has an axis of symmetry, then a circle can be inscribed in the quadrilateral.

5. Prove that if a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

From a point A on one of two straight lines OA, OD, lines are drawn to meet the other at B, C; from D lines are drawn parallel to these to meet OA at E, F. Prove that BF is parallel to CE.

6. Give and prove the construction for a mean proportional between two given lines.

A diameter AB of a circle meets a perpendicular chord

PQ at X. Prove that the circles on XA, XB, XP, XQ as diameters are together equal to the original circle.

7. Give and prove the construction for inscribing a regular hexagon in a circle.

With the vertices of a regular hexagon as centres circles are described of radius equal to one-half a side of the hexagon. Prove that *two* circles can be described to touch all six circles. What is the radius of each of the two?

8. Prove that the area of a circle is equal to one-half the product of its radius and circumference.

Prove that the areas of the circumscribed and inscribed circles of a regular hexagon are in the ratio $4:3$.

9. The radius of a circle is 12 inches in length; at the extremity of a radius a perpendicular is erected of length 10 inches, and the extremity of this is joined to the centre. Find the difference between one-half the circumference of the circle and the perimeter of the triangle thus formed.

10. How can you determine a right angle with no other instrument than a sheet of paper whose edges are true, although its angles are not right angles?

SPRING, 1909

(Two and one-half hours.)

1. Prove that the angles at the base of an isosceles triangle are equal.

Prove that if the base angles of two isosceles triangles are complementary, the vertical angles are supplementary.

2. Define a parallelogram. Prove that a diagonal of a parallelogram divides it into two triangles, equal in all respects.

A parallelogram has the diagonals of a given parallelogram as diameters, and another parallelogram has the diameters of the given parallelogram as diagonals. Prove that these two parallelograms are similar. Find the ratio that the area of each bears to the area of the given parallelogram.

3. Prove that a straight line perpendicular to a radius of a circle at its extremity is a tangent to the circle. Apply this to prove that every line at a fixed distance from a given point is a tangent to a certain circle.

AB is a diameter of a given circle, and *P* is a point on this diameter, inside the circle. Two circles are described with centers *A* and *B*, to pass through *P*. Prove that their common exterior tangents are tangents to the given circle.

4. Prove that if from a point without a circle a tangent and a secant are drawn, the tangent is a mean proportional between the whole secant and its external segment.

A point *P* moves along a fixed line; *O* is a fixed point, at a distance *OA* from the fixed line. On *OP* a point *Q* is taken, such that *OQ* is a third proportional to *OP* and *OA*. Find the locus of *Q* as *P* moves along the line.

5. Prove that the area of a parallelogram is equal to the product of its base and altitude.

Prove that the area of a triangle is the same as that of a second triangle, of which the base is equal to the perimeter of the given triangle and the altitude to the radius of the inscribed circle of the given triangle. Apply this to find the radius of the inscribed circle of a triangle whose sides are 6 feet, 8 feet, and 10 feet in length.

6. Give and prove the construction for a regular hexagon in a given circle.

AUTUMN, 1909

(Two and one-half hours.)

1. Prove that if from a point within a triangle two straight lines are drawn to the extremities of one side, their sum will be less than the sum of the other two sides of the triangle.

Prove that the sum of the distances from a point inside a triangle to the three vertices is less than the perimeter of the triangle, but greater than half the perimeter.

2. Prove that if a quadrilateral has one pair of sides equal and parallel, it is a parallelogram.

Prove that if the two parallelograms into which a given parallelogram is divided by a diameter are similar to the given parallelogram, then the sides of the given parallelogram are in the same ratio as a side diagonal of a square. What is the numerical value of this ratio?

3. Prove that the angle between two tangents is measured by one-half the difference of the intercepted arcs.

Prove that a circle can be described to pass through the four vertices of a quadrilateral whose sides are tangents to a given circle at the extremities of two perpendicular chords.

4. Prove that two triangles which are mutually equiangular are similar.

A point P moves along a fixed line; O is a fixed point not on the line. On OP a point Q is taken, which divides OP in a constant ratio. Find the locus of Q as P moves along the given line.

5. Prove that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides.

Two rectangles inscribed in a circle are equal in area. Prove that they are equal in all respects.

6. Prove that the area of a circle is equal to one-half of the product of its radius and circumference.

Find the circumference and the area of the circle through the vertices of a triangle whose sides are 5 feet, 12 feet, and 13 feet in length.

SPRING, 1910

(Two and one-half hours.)

1. Define parallel straight lines. Prove that two lines perpendicular to the same straight line are parallel.

Two parallelograms equal in all respects are placed so as to have one diagonal, but not the other, common. Prove that the remaining vertices form a rectangle.

2. Prove that angles at the centre of a circle are proportional to the arcs on which they stand.

Two circles intersect; each divides the circumference of the other in the same ratio. Prove that the circles are equal.

3. Prove that triangles which are mutually equiangular are similar.

Prove that if a triangle formed by three tangents to a circle is similar to the triangle formed by their points of contact, it is equilateral.

4. Prove that if through a point within a circle two chords are drawn, the product of the two segments of one is equal to the product of the two segments of the other.

5. Prove that the bisector of an angle of a triangle divides the opposite side into segments which are proportional to the other two sides.

The bisectors of the angles B, C meet the opposite sides at Q, R. If the lengths AQ, QC, AR, RB are 9, 15, 8, and 10 inches, find the length BC. Hence show that A is a right angle.

6. Give and prove the expression for the area of a triangle.

Prove that if the altitudes of one triangle are proportional to the sides of a second triangle, then the altitudes of the second triangle are proportional to the sides of the first.

7. Prove that if two triangles have their three sides respectively equal, they are equal.

AUTUMN, 1910

(Two and one-half hours.)

1. Prove that the perpendicular is the shortest line between a point and a straight line.

Two points P, Q are on the same side of a given straight line, at unequal distances from it. Find the shortest path from P to the line and then to Q.

2. Prove that an angle inscribed in a circle is measured by one-half its intercepted arc.

Two quadrilaterals are inscribed in equal circles, and the diagonals of one are equal to the diagonals of the other. Prove that the angles of one are equal to the angles of the other. Are the sides equal? Prove your answer.

3. Prove that if a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

Prove that the diagonals of all quadrilaterals whose sides are bisected at given points P, Q, R, S are parallel and of the same length.

4. Prove that in two similar triangles corresponding altitudes have the same ratio as any two homologous sides.

Show how to construct a triangle when the lengths of the three altitudes are given.

5. Prove that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides that contain the right angle.

6. Give and prove the construction for a regular hexagon inscribed in a given circle.

Two circles are described, one with centre A to pass through *B*, the other with centre *B* to pass through *A*. Prove that one-third of the circumference of either circle, but more than one-third of the area, lies inside the other.

7. Prove that the three lines drawn through the vertices of a triangle to bisect the opposite sides meet in one point.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO MOUNT HOLYOKE

SEPTEMBER, 1909

Omit two questions.

1. Define square; parallel lines; isosceles right triangle; median of a triangle; π; regular polygon.

If the vertical angle of an isosceles triangle is equal to 40°, find the angle included by the bisectors of the base angles.

2. Prove that the perimeters of two similar polygons have the same ratio as any pair of homologous sides.

If the side of an equilateral triangle equals the altitude of another, what is the ratio of their areas?

3. Show that the sum of the lines which join a point within a triangle to the three vertices is less than the perimeter but greater than half the perimeter.

4. Prove that a line drawn parallel to the base of a triangle divides the sides proportionally.

Find the locus of a point whose distances from two given intersecting straight lines are in a given ratio.

5. Given a circular ring included between two concentric circles. Show that a chord of the outer circle tangent to the inner circle is bisected at the point of contact, and that the area of the ring is equal to the area of a circle with this chord as diameter.

6. Derive the formula for the area of a trapezoid.

Find the side of a square equivalent to a trapezoid whose bases are 56 feet and 44 feet, and each leg of which is 10 feet.

7. Show that the line of centres of two intersecting circles bisects their common chord at right angles.

If two circles are tangent externally at a point *P*, show that the chords of the arcs intercepted on the two circles, by two straight lines drawn through *P*, are parallel. Is this theorem true, if the circles are tangent internally?

8. Show how to construct a circle equivalent to the sum of two given circles.

9. Show that the area of the star-shaped polygon formed by producing the sides of a regular hexagon is doubled that of the given hexagon.

10. Prove that two right triangles are equal if the hypotenuse and a side of the one are respectively equal to the hypotenuse and a side of the other.

If two altitudes of a triangle are equal, show that the triangle is isosceles.

SEPTEMBER, 1910

1. Define isosceles triangle. Show that if one side of an isosceles triangle is extended through the vertex, the bisector of the external angle so formed is parallel to the base.

2. Show that the radius of a circle drawn perpendicular to a chord bisects the chord. Find the locus of the points of bisection of parallel chords of a circle.

3. Under what conditions may a parallelogram be inscribed in a circle.

Find the conditions under which a quadrilateral may be circumscribed about a circle.

4. State and prove the expression for the area of a trapezoid. Find the side of a square equivalent to a trape-

zoid whose bases are 56 feet and 44 feet, and whose legs are each 10 feet.

5. Define similar figures. Prove that if the areas of two similar figures are equal, the perimeters are equal.

6. State and prove the relation between the sides of a right triangle.

Show how to construct a circle equal in area to the sum of two given circles.

7. Fine the relation between the lengths of the side of a square and of its diagonal.

With a given length as unit show how to construct lines of length $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, respectively.

8. Two equal chords intersect within a circle. Prove that the two segments of one are equal respectively to the two segments of the other.

9. Show how to construct a mean proportional to two given lines.

Show that the line bisecting the vertex angle of a triangle divides the base into segments proportional to the sides.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO SMITH

SEPTEMBER, 1909

(Students are advised to answer the questions in order, but time may be saved by leaving the harder questions for the last. In the solution of problems of construction, full proof is to be given.)

1. In the same circle or in equal circles, two central angles have the same ratio as their intercepted arcs. Prove only Case II where the arcs are incommensurable.

2. The non-parallel sides of a trapezoid and the line joining the middle points of the parallel sides, if produced, pass through the same point.

3. If from a fixed point without a circle a secant is drawn, the product of the secant and the external segment is constant, in whatever direction the secant is drawn.

4. The areas of two triangles which have an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles.

5. Problem: to construct a square having a given ratio to a given square.

6. If in a circle two chords intersect at right angles and circles are constructed on the segments of the chords as diameters, the area of the given circle is equivalent to the sum of the areas of the four circles.

7. Problem: to inscribe a regular decagon in a circle.

SEPTEMBER, 1910

(*Students are advised to answer the questions in order, but time may be saved by leaving the harder questions for the last. In the solution of problems of construction full proof must be given.*)

1. Given angle B and sides a and b; construct the triangle. Discuss all possible cases.

2. The bisector of an interior angle of a triangle divides the opposits side into segments proportional to the other two sides.

3. The areas of two rectangles having equal altitudes are to each other as their bases, when the bases are incommensurable.

4. Given the radius r and the side s of a regular inscribed polygon. Find the side of a regular inscribed polygon of double the number of sides.

Answer two of the following. No extra credit will be given for more than six questions.

1. If two circles intersect, their common chord produced bisects their common tangent.

2. If the sides *AB* and *BC* of an inscribed quadrilateral *ABCD* subtend arcs of 69° and 112° respectively and the angle *AED* between the diagonals is 87°, how many degrees are there in each angle of the quadrilateral?

3. Define the term trapezoid.

The two parallel sides of a circumscribed trapezoid are 6 feet and 18 feet respectively and the other two sides are equal. Find the radius of the circle.

EXAMINATIONS IN PLANE GEOMETRY FOR ADMISSION TO WELLESLEY

SEPTEMBER, 1909

Answer six questions.

1. Define the terms: quadrilateral, locus, regular hexagon. When are figures equivalent? When are they similar?

2. State three cases in which triangles are similar, and prove one of them.

3. Show how a given line may be divided in extreme and mean ratio. What is the relation between the line and the segments into which it is divided.

4. If, from any point in the plane of a circle, a tangent and a secant are drawn, the product of the whole secant and its external segment is equal to the square on the tangent.

5. An angle formed by two tangents to a circle is how measured? Prove.

6. Find the area of a circle inscribed in an equilateral triangle the length of whose side is $8\sqrt{3}$.

7. The area of a parallelogram is 96 square feet. The points *A* and *B* trisect one side; the point *C* bisects the opposite side. Find the areas of the three parts into which the parallelogram is divided by *AC* and *BC*.

8. If a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

SEPTEMBER, 1910

1. If two parallel lines be cut by a transversal, the alternate-interior angles are equal.

2. Find the area of an equilateral triangle circumscribed about a circle of radius 5.

3. What is the locus of points equidistant from the extremities of a straight line? Prove.

4. If two triangles have their homologous sides proportional, they are similar.

5. An angle formed by a tangent and a chord drawn from the point of contact is measured by one-half the intercepted arc.

6. State the formulæ for the circumference and area of a circle. Prove one.

7. Two circles, *O* and *O′*, intersect in *A* and *B*. *C* is the extremity of the line of centres on circle *O*. *CA* and *CB* cut circle *O′* in *D* and *E*. Prove triangles *ABD* and *ABE* equal.

Plane and Solid Geometry

By

FLETCHER DURELL, Ph.D.

Head of the Mathematical Department
The Lawrenceville School

ONE of the main purposes in writing this book has been to try to present the subject of geometry so that the pupil shall understand it, not merely as a series of correct deductions, but shall realize the value and meaning of its principles as well. This aspect of the subject has been directly presented in some places, and it is hoped that it pervades and shapes the presentation in all places.

Again, teachers of geometry generally agree that the most difficult part of their work lies in developing in pupils the power to work original exercises. The second main purpose of the book is to aid in the solution of this difficulty by arranging original exercises in groups, each of the earlier groups to be worked by a distinct method. The pupil is to be kept working at each of these groups until he masters the method involved in it. Later, groups of mixed exercises to be worked by various methods are given.

In the current exercises at the bottom of the page, only such exercises are used as can readily be solved in connection with the daily work. All difficult originals are included in the groups of exercises as indicated above.

Similarly, in the writer's opinion, many of the numerical applications of geometry call for special methods of solution, and the thorough treatment of such exercises should be taken up separately and systematically. In the daily extemporaneous work only such numerical problems are included as are needed to make clear and definite the meaning and value of the geometric principles considered.

Every attempt has been made to create

and cultivate the heuristic attitude on the part of the pupil. This has been done by the method of initiating the pupil into original work described above, by queries in the course of proofs and at the bottom of different pages, and by occasional queries in the course of the text where definitions and discussions are presented. In the writer's opinion, the time has not yet come for the purely heuristic study of geometry in most schools, but it is all-important to use every means to arouse in the pupil the attitude and energy of original investigation in the study of the subject.

In other respects, the aim has been to depart as little as possible from the methods most generally used at present in teaching geometry.

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